1. A *ternary* linear \((n,k,d)\)-code is a subspace of \(\mathbb{Z}_3^n\) — in other words, a set of strings of \(n\) letters from \(\mathbb{Z}_3 = \{0,1,2\}\) which is closed under component-wise addition modulo 3.

Let \(C\) be the ternary linear code with generator matrix

\[
G = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 2 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 2 & 2 \\
0 & 0 & 0 & 1 & 0 & 0 & 2 & 1 & 0 & 1 & 2 \\
0 & 0 & 0 & 0 & 1 & 0 & 2 & 2 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & 2 & 1 & 0
\end{pmatrix}.
\]

Evaluate \(n\) and \(k\), and show that \(d \leq 5\). If you’re in the mood for it, show that \(d\) is equal to 5. Hence find the rate and error detection/correction properties of \(C\).

2. Let \(C\) be the Hamming code with parity check matrix

\[
H = \begin{pmatrix}
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 & 0
\end{pmatrix}.
\]

Suppose that a codeword \(c = 1110000 \in C\) was transmitted 3 times, and that 1 error occurred during each transmission, so that the received vectors were \(r_1 = 0110000\), \(r_2 = 1111000\) and \(r_3 = 1110001\).

Compute \(Hr_i^T\) for each \(i\). What do you notice about the results?

3. Calculate the out-of-phase autocorrelation \(AC(k)\) of the sequence with period 5 and pattern 01011 for \(k = 1, 2, 3, 4\). Does the sequence satisfy Golomb’s third postulate G3?

4. Draw a circuit diagram for the LFSR with characteristic polynomial \(x^6 + x^2 + 1\). What output is produced with seed 100000?