Tutorial Questions

Submit your solutions to ALL Tutorial Question to your Tutor's assignment box by 2:30 pm, Friday 10 August. Make sure you attach a cover sheet (available on line) to your solutions.

1. An economist determines the following table for next year’s Australia wide inflation rate (rounded to whole numbers) with the corresponding probabilities of occurrence:

<table>
<thead>
<tr>
<th>Rate (%)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.20</td>
<td>0.24</td>
<td>c</td>
<td>0.19</td>
<td>0.10</td>
<td>0.06</td>
</tr>
</tbody>
</table>

(a) Define the random variable, say \( X \), of interest to the economist.
(b) Find the value of \( c \).
(c) Tabulate, using appropriate symbols, the p.m.f. of \( X \).
(d) Sketch a graph of the cumulative distribution function of \( X \).
(e) Calculate
   i. \( P(X = 1) \)
   ii. \( P(X \geq 2) \)
   iii. \( P(X < 5) \).
(f) Find \( E(X) \) and describe in words what this tells you.
(g) Find \( E(X - 1.93)^2 \).

2. The probability that a machine breaks down at most once in a month is 0.9, and the probability it breaks down at least once in a month is 0.2. Calculate the probability that the number of breakdowns in a month is
   i. 0 , ii. 1 , iii. at least 2.

3. A retailer has shelf space for three units of a highly perishable item that must be disposed of at the end of the day if not sold. Each unit costs $3.00 and sells for $5.00. Demand probabilities are as follows:
   \( P(\text{Demand} = 0) = 0.30, P(\text{Demand} = 1) = 0.45, P(\text{Demand} = 2) = 0.25 \). Let \( X \) be a random variable indicating daily profit if the retailer stocks two units each day. Let \( Y \) be a random variable indicating daily profit if the retailer stocks one unit each day.

   (a) Find the probability distributions for \( X \) and \( Y \).
   (b) Using the expected values of \( X \) and \( Y \), determine whether the retailer would be better off stocking one or two units per day.

Practice Questions

1. A business marketing flower bulbs knows that 98% of its bulbs will flower. They are sold in packets of 5 randomly selected bulbs with a guarantee that the packet will be replaced if 100% flowering is not achieved.

   (a) What is the probability that it will be necessary to replace a given packet under this guarantee? Interpret this probability.
   (b) What would be the probability of replacing a packet if the guarantee covered only at least 4 out of 5 bulbs flowering? Comment on your findings in parts (a) and (b).

2. Determine if the following are true or false. Provide reasons for your answers.

   (a) The mean of a discrete random variable is always positive.
(b) If the mean of a random variable is positive then the random variable only takes on positive values.

3. Decide which of the following functions could represent probability mass functions for a random variable $X$. For those which could, write out the table for the probability mass function and determine $E(X)$. Also draw the graph of the cumulative distribution function for the first of them.

(a) $p(x) = \frac{x}{10}$, where $x = 1, 2, 3, 4$;
(b) $p(x) = 0.7x$, where $x = -1, 0, 1, 2$;
(c) $p(x) = \frac{x^2}{14}$, where $x = 0, 1, 2, 3$;
(d) $p(x) = \frac{1}{x}$, where $x = 1, 2, 3$;
(e) $p(x) = (10 - x)/40$, where $x = 0, 1, 2, 3, 4$.

4. A motor vehicle inspection unit collected the following data during the past year. Assuming the observed pattern continues to hold, what would be the unit’s expected daily revenue this year if it charges $25 per car?

<table>
<thead>
<tr>
<th>Number of cars inspected per day ($x_i$)</th>
<th>20</th>
<th>25</th>
<th>32</th>
<th>40</th>
<th>47</th>
<th>51</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of days ($f_i$)</td>
<td>79</td>
<td>121</td>
<td>19</td>
<td>25</td>
<td>39</td>
<td>30</td>
</tr>
</tbody>
</table>

5. A discrete random variable $X$ has the following p.m.f.:

\[
\begin{array}{c|cccc}
 x & 1 & 2 & 3 & 4 \\
p_X(x) & 0.1 & 0.2 & 0.1 & 0.6 \\
\end{array}
\]

(a) Evaluate $E(X)$, $\text{Var}(X)$ and $E(X^3 - 1)$.
(b) Sketch the graph of the cumulative distribution function for $X$.
(c) Define a new random variable $Y$ as follows: $Y = 0$ if $X > 2$ and $Y = 1$ if $X \leq 2$. Determine the p.m.f. of $Y$. What is the distribution of $Y$?

6. A discrete random variable $X$ takes the values 1, 4 and 6, with probabilities 0.4, 0.5 and 0.1, respectively. Calculate $E(X)$ and $E(X^2 - 2X)$.

7. Among 4 market analysts, the number of correct predictions $N$ has probability distribution given by

\[
P(N = n) = \frac{9 - 2n}{25}, \quad n = 0, 1, 2, 3, 4.
\]

Write down the p.m.f. table for the random variable $N$, and use it to find

i. $P(N > 2)$, \hspace{1cm} ii. $P(N \leq 1)$, \hspace{1cm} iii. $E(N)$.

8. The table below shows the probability mass function of a random variable $X$, where $c$ is a constant.

\[
\begin{array}{c|cccc}
 x & 0 & 1 & 2 & 3 \\
P_X(x) & c & c^2 & c^2 + c & 3c^3 + 2c \\
\end{array}
\]

(a) What is the value of $c$?
(b) Using this value of $c$, find $E(X)$.

9. **Black et al.** 5.3, 5.10, 5.13, 5.14

10. For each random variable described below state whether a Binomial random variable would provide a satisfactory model. Give reasons to support your decision and where appropriate state values for $n$ and $p$.

(a) The number of shares in the ASX Top 500 that will fall in value today.
(b) The number of new businesses that will fail this year.
(c) The number of interest rises by the Reserve Bank in a calendar year. (Note that the Reserve Bank considers interest rate rises every month.)
(d) The number of houses from his listing of 25 that a real estate agent will sell in a month.