Test 4 — Mock Mid-Semester Test

Student Number: [ ]

Family Name: ________________________________

Use Block Letters

Given Name: ________________________________

Use Block Letters

Lecture Stream: ________________________________

8 am or 9 am

This test counts for 15% of your final mark for MATH1050 Calculus C.

There are FOUR questions in this test, worth a total of 50 marks.

Attempt all questions.

Marks are given for clarity and correctness of method, not just for correct answers.

Appropriate working with reasons should be given.

Write your answers in the blank space following the question.

The last page has been left blank for rough working.

Time Limit: 50 minutes
1. **Linear Equations (11 marks)**

(a) Solve the following equation for $x$. 

$$\frac{x - 3}{3} + 1 = \frac{x + 3}{6}$$
(b) How many litres of a 80% hydrochloric acid solution should be added to a 40% hydrochloric acid solution to make 4 litres of a 50% solution? (4 marks)
(c) A landscaping company placed two orders with a nursery. The first order was for thirteen bushes and four trees costing a total of $487. The second order was for six bushes and two trees costing a total of $232. The bill did not list the cost of each item. What was the cost of each bush and tree? (4 marks)
2. Factorisations and Quadratics (11 marks)

(a) Simplify

\[
\frac{6x^2 + 7x + 2}{4x^2 - 1} \div \frac{9x^2 + 18x + 8}{9x^2 - 16} \times \frac{4x^3 - x}{3x^2 - 4x}
\]

(5 marks)
(b) For what value(s) of \( k \) does the equation \( 36x^2 - 6kx + 1 = 0 \) have exactly one solution? 

(2 marks)

(c) A missile is fired up in the air at the edge of a 1000 m high cliff so the height \( h(t) \) of the missile \textbf{above the cliff} at time \( t \) (in seconds) is given by

\[
h(t) = 50t - 5t^2, \quad t \geq 0.
\]

The missile travels up in the air and then falls on the beach below. How long does the missile take to land on the beach below? 

(4 marks)
3. Exponents and Logarithms (16 marks)

(a) Simplify the following expressing with positive indices:

\[ \left( \frac{9x^2}{y^{-4}} \right)^{\frac{1}{2}} \div \sqrt[4]{\frac{16x^8}{81y^{-4}}} \]

(4 marks)
(b) Let \( x = \log_3 4 \) and \( y = \log_3 5 \). Express the following in terms of \( x \) and \( y \).

i. \( \log_3 20 \) \( \quad \text{(2 marks)} \)

ii. \( \log_3 0.75 \) \( \quad \text{(3 marks)} \)

iii. \( \log_5 48 \) \( \quad \text{(3 marks)} \)
(c) Solve the following equation for \( x \):

\[
3^{x+1} = 5^{2x-1}
\]

(4 marks)
4. Functions (12 marks)

(a) Determine if \( f(x) = \log(1 - x) \) is a function, state its domain and range sketch its graph. (3 marks)
(b) Let \( h(x) = x^2 - 5x + 6 \). Sketch the graph of \( h(x) \), showing all intercepts and the co-ordinates of the vertex. (4 marks)
(c) Let \( y = x^2 - 4x + 6 \).

i. Show that \( y - 2 = (x - 2)^2 \) \hspace{1cm} (2 marks)

ii. Hence sketch the graph of \( y \), showing any intercepts and the co-ordinates of the vertex. \hspace{1cm} (3 marks)