1. Consider the two polynomials:
   \[ A(x) = 10x + 5 \]
   \[ B(x) = 2x + 1 \]
   (a) Calculate \( C(x) = A(x)B(x) \) by direct algebraic multiplication? What is the time complexity as a function of the polynomial degree \( n-1 \)?
   (b) Carry out the multiplication by converting to point-value representation based on the complex roots if unity.
   (c) Show how the FFT can be used in (b) and what is the time complexity for (b) as a function of the polynomial degree \( n-1 \) if the FFT is used?

2. A relative frequency count of the first seven letters of the English alphabet reveal the following statistics:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>5</td>
<td>10</td>
<td>13</td>
<td>30</td>
<td>10</td>
<td>7</td>
</tr>
</tbody>
</table>

   (a) Derive the optimum binary code by building the Huffman coding tree
   (b) Show how to encode the sequence: AAEFDEEG
   (c) Show how to decode the sequence: 001000001010111111101

3. Here are some questions on analysis and design:
   (a) Compare and contrast the different sorting algorithms for sorting a list of real numbers efficiently.
   (b) What is the difference between a red-black tree and an optimal binary search tree? Does one imply the other? Can a red-black tree also be an optimal BST?
   (c) Which collision resolution scheme for hash tables would you choose? Why?
   (d) Is finding the minimum spanning tree the same as finding the shortest distance between any two nodes? Can a MST also be a shortest distance tree (between any two nodes)?
   (e) Are you sure finding the Hamiltonian path is a class NP problem?