1. Introduction to Adaptive Systems

1.1 Random Signals

1.1.1 What is a signal?
Denote the discrete-time signal \( x(n) \) as a sequence of ordered samples (for each \( n \)).
Possible sources include:
- Sampling a continuous-time signal, \( x(n) = x_c(nT) \), where \( T = \frac{1}{F_s} \) is the sampling period and \( F_s \) is the sampling frequency,
- Sampling the output of an array of sensors at time \( t \), \( x(n) = x_n(t) \), for each sensor \( n \).

1.1.2 What is randomness?
Successive observations, \( x(n-1), x(n), x(n+1), \ldots \) are dependent on one another
- If \( x(n) \) can be predicted exactly from the previous samples then the time-series signal is deterministic
- If \( x(n) \) cannot be predicted exactly from the previous samples the time-series signal is random or stochastic
- If \( x(n) \) is independent from previous samples then the time-series signal is white noise.

1.1.3 What do we want to do with these signals?
Spectral estimation – extract features useful for classification
Signal modeling – understanding the underlying signal generation process
Signal filtering – improve quality of signal according to a criterion of performance

1.2 Spectral Estimation

1.2.1 Single-signal amplitude analysis
Examines the statistical distribution of the signal amplitudes.
Measurements of interest include:
- mean, median and variance
- probability distribution function
- dynamic range of amplitude

1.2.2 Single-signal dependence analysis
Examines the amount of correlation between samples of the same signal.
Measurements of interest include:
- autocorrelation and power spectrum
- self-similarity and higher-order statistics
1.2.3 Joint-signal analysis
Simultaneously examines two signals for their inter-dependence and inter-relationship.
Measurements of interest include:
- cross-correlation and cross-power spectrum
- coherence and higher-order statistics

1.3 Signal Modeling

1.3.1 What is signal modelling?
We want to derive a signal model which is a mathematical description that provides an efficient representation of the “essential” properties of the signal.
- a model can be used to generate random signals with the “essential” properties
- a model can be used to describe a particular class of signals and form the basis for classification comparing the “essential” properties and ignoring the irrelevant properties
- a model can be used for efficient storage and transmission of the signal

Efficient representation is achieved by:
- selecting a “good” model
- selection of the “right” number of parameters
- achieving a good fit of the model to the data with $M << N$, where $M$ is the number of parameters and $N$ is the number of data samples.

Essential properties of the signal include:
- identical magnitude spectrum for selected frequencies or all frequencies
- acceptable approximation of $x(n)$, especially when this is based on previous samples of the signal.

1.3.2 Pole-Zero Models
One of the most popular modelling paradigms is the pole-zero model which models $x(n)$ based on the previous samples of the signal, $x(n-k)$, and the current and past values of an external excitation source, $w(n-k)$:

$$x(n) = \sum_{k=1}^{p} (-a_k) x(n-k) + \sum_{k=0}^{Q} d_k w(n-k)$$

Equation 1.1

The resulting transfer function:

$$H(z) = \frac{X(z)}{W(z)} = \frac{\sum_{k=0}^{Q} d_k z^{-k}}{1 + \sum_{k=1}^{p} a_k z^{-k}}$$

Equation 1.2

is known as the pole-zero model of the time-series signal.
Other names include:
- AutoRegressive Moving-Average (ARMA) model
- AutoRegressive (AR) model for the special case of $Q = 0$ (all-pole model)
- Moving Average (MA) model for the special case of $P = 0$ (all-zero model)

1.4 Adaptive Filtering

1.4.1 What is adaptive filtering?

Characteristics of adaptive filters:
- filter response modifies itself (“adapts”) according to the non-stationary, noisy or otherwise unpredictable behaviour of the signal.
- filter co-efficients are recalculated/re-estimated constantly, rather than being fixed at the initial design stage (as is the case with traditional digital filters).
- three modules of design: the filter structure itself, estimating the error criterion of performance, adapting the filter (i.e. recalculating the filter co-efficients)

1.4.2 Application to System Identification

*Block Diagram*
Adaptive filter provides an estimate of the desired signal or system response

![System Identification Diagram](Figure 1.15(a) [1])

**Examples**
- **echo cancellation**: remove undesirable echo from two-way communications system by subtracting an estimate of the echo
- **adaptive control**: estimate parameters and/or state of the plant in order to design a controller
- **channel modeling**: provide estimate of the unknown system response over the regions of operation of interest
Specific Example of Acoustic Echo Cancellation

Problem description: in two-way communications (telephone, audio teleconferencing, etc.) the transmitting microphone will pick up unwanted sound from the receiving loudspeaker which is transmitted back to the original speaker as an echo (with a delay equal to the round-trip transmission time).

Problem solution: adaptive filter forms an estimate of the interfering signal picked up by the microphone, \( e(t) \), given the incoming loudspeaker signal, \( x(t) \), and subtracts this from the outgoing microphone signal, \( y(t) = s(t) + e(t) \), so that only the wanted signal, \( s(t) \) is transmitted.

Solution caveats:
1. The echo is not just a simple, linear function of the loudspeaker signal. Room acoustics and speaker and microphone transducer effects will modify the signal echo in a complex way.
2. The room acoustics, and to a lesser extent transducer effects, change with time as the talker moves, microphone/speaker are re-positioned, etc.

1.4.3 Application to System Inversion

Block Diagram
Adaptive filters provides an estimate and applies the inverse of the system response
Examples

• **adaptive equalization**: apply inverse of communication channel transfer function in order to equalise or remove the unwanted effects or distortions arising from transmission through the channel.

• **blind deconvolution/separation**: apply the inverse of the corrupting convolution or mixing operations to the resultant output signal(s) in order to reconstitute the original signal. The system convolution/mixing response is usually unknown and this makes the problem “blind”.

• **adaptive inverse control**: estimate the inverse response of the plant in order to design controllers in series rather than in feedback with the plant.

Specific Example of Channel Equalization

**Problem description**: the detector in a digital communications system has to determine whether the received pulse is symbol 1 or symbol 0 every \( T_b \) seconds. The pulse is designed so that there are zero-crossings every \( T_b \) seconds in order to avoid interference with adjacent pulses. However this relies on transmission through a distortion-less or ideal channel and real-world channels will introduce amplitude and phase effects that will distort the shape of the pulse and produce **InterSymbol Interference (ISI)**

**Problem solution**: compensate for ISI by using an adaptive filter that restores the received pulse to the original shape by estimating and applying the inverse of the communication channel response characteristics.

![Figure 1-4 Channel equalizer with training and decision-directed modes of operation](image)

**Solution caveats**

1. The filter has to both “learn” the inverse of the particular channel and “track” its variation with time

2. Requires knowledge of the correct symbol sequence (e.g. a training sequence) initially to prime or reset the operation of the filter. In cases where this is not available then there is the additional problem of “blind equalization”.

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1.4.4 Signal Prediction

**Block Diagram**
Estimate a signal at time $t = n_0$ by using past and/or future values of the same signal \{x(t): n_1 \leq t \leq n_2\}
- forward prediction (linear prediction) if $n_0 > n_2$
- backward prediction of $n_0 < n_1$
- smoothing/interpolation if $n_1 < n_0 < n_2$ (excluding $n_0$ itself!)

![Signal Prediction Block Diagram](image)

**Figure 1-5** Signal Prediction (Figure 1.15(c) [1])

**Examples**
- adaptive predictive coding: form estimate of current value of signal based on previous samples and store/transmit the error in the prediction rather than the signal itself

**Specific Example of Linear Predictive Coding (LPC)**

**Problem description:** time-series data from real-world sources like speech are highly correlated. When storing or transmitting such data quantisation involves representing the dynamic range of the signal amplitude over a discrete range of values, the larger the dynamic range the larger the discretisation steps or errors (i.e. quantisation noise) for the same bit quantisation of the signal.

**Problem solution:** to reduce the quantisation noise the current sample is predicted based on the previous samples and the error in the prediction is stored/transmitted together with the predictor filter co-efficients. The dynamic range of these quantities should be smaller and hence subject to reduced quantisation noise effects.
Solution caveats
1. As real-word sources are non-stationary the filter has to recalculate the predictor coefficients and retransmit these to the receiver, but this is done on a frame-by-frame rather than sample-by-sample basis.

1.4.5 Multisensor Interference Cancellation

Block Diagram
Use of multiple sensors that provide reference signals for the estimate and removal of interference and noise from a primary signal.

Examples
- **active noise control**: provide an inverted estimate of the unwanted signal or noise and remove it from the zone of interest by destructive wave interference
- **array processing**: collect signals from a group of spatially positioned sensors and emphasise signals arriving from specific directions (i.e. adaptive beamforming) as used in radar, direction finding, antenna steering, etc.
Specific Example of Active Noise Control (ANC)

**Problem description**: in noisy environments (e.g. airline cockpit, car) interfering signals and noise make listening difficult and uncomfortable.

**Problem solution**: a reference signal, \( x(t) = G_s v(t) \), from the interfering environment is used to provide an exact out-of-phase estimate of the interfering signal(s), \( y(t) = G_y v(t) \), that is \( \hat{y}(t) = f_x(x(t)) = G_y G_s^{-1} x(t) \), which when played out through a loudspeaker will completely cancel the unwanted signal(s) in the listening zone (i.e. \( y(t) + \hat{y}(t) = 0 \)).

**Solution Caveats**

1. The filter has to provide an exact out-of-phase estimate of the interfering signal in order to cancel the unwanted signal, otherwise more noise is added!
2. The reference signal must be a filtered version of the interfering signal but must not include any desired signals in the listening zone of interest, otherwise these will also be removed.
3. The acoustic environment is unknown and highly time-varying and the filter must be able to rapidly adapt to any changes.

1.5 **References**