1. (a) Write the MATLAB code for the Kalman filter equations given the following header definition (note that $Q \equiv R_u$ and $R = R_v$):

```matlab
function [y, Py, Kc] = mykalman(x,A,B,Q,H,R,y0,P0)
% Performs Kalman filter for time-invariant systems
% [Y,PY,KC] = MYKALMAN(X,A,B,Q,H,R,Y0,P0) returns
% Y, the K-by-N filter state sequence
% PY, the K-by-K final state error (MMSE) covariance
% KC, the M-by-K final (steady-state) Kalman gain matrix
% given:
% X the M-by-N observed data sequence
% A,B,Q the state model system matrices
% H,R the observation model system matrices
% Y0 the initial state
% P0 the initial state error covariance
```

(b) Write the MATLAB code using the `mykalman` function in (a) to generate estimates of the desired AR(2) signal, $y(n)$, in Example 3-5 given 100 samples of observation data, $x(n)$ and plot the desired signal, its estimate and the observation data.

2. Consider the model of a digital data communication system shown below:

```
Delay
D

a(n) ----> Channel h(n) ----> Equalizer c(n) ----> e(n)
        |                          |
        |                          v(n)
        |                          |
        |        x(n)             |
        |                          |
        |                          |
```

The input symbol sequence is a string of independent, random binary digits \{0,1\} with covariance $C_a$. The communications channel has an impulse response, $h(n) = 0, \quad n \geq L$ for some integer $L$ and the channel is subject to additive white noise, $v(n) \sim N(0, \sigma^2)$. An equalizer is required to combat the effect of the ISI and additive white noise to produce an time-delayed estimate of the input to the channel, $\hat{y}(n) = a(n-D)$, where $D$ represents the input delay through the system. Example 6.8.1[1] illustrates the optimum MMSE solution to the problem. Here we will attempt an alternative MMSE solution based on the Kalman filter as follows:

(a) Derive the expression for $x(n)$ as a function of the noise, $v(n)$, channel impulse response, $h(n)$, current input, $a(n)$, and channel memory, $a(n-1), a(n-2), \ldots, a(n-L)$. Identify the desired signal, ISI and additive white noise interference.

(b) Formulate the observation process equation and hence identify what the state vector of the system is.

(c) Formulate a corresponding state process equation remembering that $a(n)$ is a random process.

(d) Hence state the solution to the problem via the Kalman filter by identifying the system matrices and estimate $\hat{y}(n)$

References