1. The Levinson-Durbin algorithm is used to derive the best AR($p$) signal model from a set of available typical observations \{\(x(n)\)\}_{n=0}^{N-1} of the signal.
   (a) Explain how the Levinson-Durbin algorithm is executed if only the data \{\(x(n)\)\}_{n=0}^{N-1} is available?
   (b) Explain how the order $p$ of the AR($p$) is selected?
   (c) State the AR($p$) description of the signal and the relevant parameters that need to be calculated from the Levinson-Durbin algorithm?
   (d) Explain how you would evaluate the model using the autocorrelation test?

2. Smoothing contiguous values of the periodogram leads to the Blackman-Tukey approach as follows:
   (a) Consider the smoothing operation:
   \[
   \hat{R}_k^{(PS)}(e^{j\omega_k}) = \frac{1}{2M+1} \sum_{j=-M}^{M} \hat{R}_k(e^{j\omega_{k-j}})
   \]
   where \(\omega_k = 2\pi k / N\). Assuming the samples of the periodogram are mainly uncorrelated show how this smoothing reduces the variability of \(\hat{R}_k(e^{j\omega_k})\) and what the tradeoff is.
   (b) Show that the smoothing operation in (a) is equivalent to convolution of \(\hat{R}_k(e^{j\omega_k})\) with a window function, \(W(e^{j\omega_k})\).
   (c) Since convolution in the spectral domain is multiplication in the time domain show how the smoothing operation in (a) can be implemented more efficiently by multiplying the sampled autocorrelation sequence with an appropriate window function. This is the Blackman-Tukey algorithm.

3. Consider the LS all-pole parametric method for spectral estimation:
   \[
   \hat{R}_k(e^{j\omega_k}) = \sigma_w^2 \left| \frac{1}{1 + \sum_{k=1}^{p} \hat{\alpha}_k e^{-j\omega_k}} \right|^2
   \]
   (a) How are the \(\{\hat{\alpha}_k\}_{k=1}^{p}\) co-efficients and \(\sigma_w^2\) calculated given \(\{x(n)\}_{n=0}^{N-1}\)?
   (b) The LS all-pole method assumes that the prediction error (residual) is white (i.e. perfect model fit). By considering the $z$-transform of the AR($P$) process:
   \[
   e(n) = x(n) + \sum_{k=1}^{p} \hat{\alpha}_k x(n-k)
   \]
   derive an expression for \(\hat{R}_k(e^{j\omega_k})\) that does not require the residual to be white.