Performance Preserving Controller Reduction via Additive Perturbation of the Closed-Loop Transfer Function

G. Wang, V. Sreeram, and W. Q. Liu

Abstract—In this note, a new \( H_{\infty} \) performance preserving controller order reduction method is proposed. Here performance preservation indicates that the \( H_{\infty} \) norm bound of the closed loop transfer function with reduced-order controller is not greater than the \( H_{\infty} \) norm bound of the closed loop transfer function with full order controller. We assume additive perturbations to the closed-loop transfer function and obtain a sufficient condition for performance preservation. Two kinds of useful weightings are derived, and the controller reduction problem is solved via a frequency weighted model reduction problem.

Index Terms—Controller reduction, model reduction.

I. INTRODUCTION

Controller order reduction has been extensively studied for more than a decade. Study indicates that the preferable controller reduction methods are those in which the information of closed-loop system has been used in the reduction algorithms. In [3], Anderson and Liu described general concepts and approaches for controller reduction and they suggest that several criteria should be considered, one of which is closed-loop \( H_{\infty} \) performance criterion. In recent years, many reduction approaches have been developed based on closed-loop \( H_{\infty} \) performance criterion [7], [9], [11]. Numerical examples show that these approaches are much better than those methods that have been developed based on closed-loop stability criterion.

The controller reduction methods that incorporate the closed-loop \( H_{\infty} \) performance can be roughly classified into two categories:

The first category includes methods aiming to minimize the error between the closed-loop system with the full order controller, and the closed-loop system with the reduced-order controller. This category can be further classified into two subclasses. The first subclass converts the error between the closed-loop system with the full order controller and the closed-loop system with the reduced-order controller to the error between the full order controller and the reduced-order controller with a pair of frequency weightings and then use usual frequency weighted model reduction methods to reduce the controller order [3]. There are many frequency weighted model reduction methods available including [4], [5], [8], [6]. The second subclass includes methods which use controllability and observability Gramians of the closed-loop systems, such as structurally balanced controller order reduction methods [7], [9].

The second category is called performance preserving controller reduction. Here, performance preservation implies that the \( H_{\infty} \) norm 'bound' of the closed loop transfer function with the reduced-order controller is not greater than the \( H_{\infty} \) norm 'bound' of the closed-loop transfer function with the full-order controller. Lenz [10] proposed a method in which information from the closed-loop system is used in frequency weightings. However, with this method the \( H_{\infty} \) performance of the closed-loop system may degrade, as pointed out in [11]. Goddard and Glover presented another performance preserving controller reduction method, which improves Lenz’ method and also guarantees the \( H_{\infty} \) performance of the closed-loop system under certain conditions.

In this note, we study the performance preserving controller reduction problem. Instead of using additive perturbation on the controller, proposed by Goddard and Glover, our method is based on additive perturbation on the closed-loop transfer function. Compared with Goddard and Glover’s method, our method has two advantages: first the orders of the proposed weightings in our method are much lower; second our method can be used with many existing controller reduction methods [7], [9], [12] since there is a close relationship between our method and the existing methods.

II. PRELIMINARIES

Consider the closed-loop system in Fig. 1 with external input \( w \), controlled output \( z \), control input \( u \), and measured output \( y \). The plant \( P \) and the full-order controller \( K \) are given by

\[
P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix},
\]

\[
K = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix}.
\]

Assuming that the inverse of \( (I - P_{22} K) \) exists, the closed-loop system \( G \) is given by the lower linear fractional transformation (LFT):

\[
G = \mathcal{F}_1(P, K) = P_{11} + P_{12} K (I - P_{22} K)^{-1} P_{21}.
\]

In practice, it is desirable to design a controller \( K \) that stabilizes \( P \) and at the same time, makes \( \| \mathcal{F}_1(P, K) \|_{\infty} \) less than a given criterion \( \gamma (\gamma > 0) \).

Definition II.1: A controller \( K \) is said to be \((P, \gamma)\)-admissible if \( K \) stabilizes \( P \) and \( \| \mathcal{F}_1(P, K) \|_{\infty} < \gamma \). In this note, we can make the assumption, without loss of generality, that the plant \( P \) has been scaled such that

\[
\| \mathcal{F}_1(P, K) \|_{\infty} < \gamma = 1.
\]

In order to reduce the controller order and at the same time keep the performance of the system within an acceptable range, any controller order reduction method should take the closed loop performance into account. Suppose \( G_r \) is the reduced-order controller, then the closed-loop system becomes \( \mathcal{F}_1(P, K_r) \), which is further denoted by \( G_r \). Based on the performance of \( G_r \), the following three criteria could be used when reducing the controller order:

1) \( G_r \) is stable;
2) \( \| G - G_r \|_{\infty} \) is small;
3) \( \| G_r \|_{\infty} < 1 \) if \( \| G \|_{\infty} < 1 \);

0018-9286/01$10.00 ©2001 IEEE
Using the different criteria, different categories of controller reduction methods are derived. There are many papers [4], [3] which study the first two categories, especially, the second one which include several different approaches [7], [9]. The third category, which was proposed by Goddard and Glover [11], is called performance preserving controller reduction (PPCR).

III. NEW CONTROLLER REDUCTION METHOD

In this section, a new controller-order reduction method is proposed. This method is based on additive perturbation of the closed-loop transfer function.

Consider a class of closed-loop transfer functions with reduced-order controller which can be represented as

\[ G_r = G - W_2 \Delta W_1 \]

where \( G \) is the closed loop transfer functions with full order controller, \( \Delta \) is a stable perturbation, \( W_1 \) and \( W_2 \) are stable, invertible weighting functions with minimum phase. Obviously this kind of perturbation is important to the study of controller reduction since the order reduction of the controller is reflected in the closed-loop transfer function. We can rewrite the above equation as

\[ \Delta = W_2^{-1}(G - G_r)W_1^{-1}. \]

The above formula shows that \( G_r \) or \( K_r \) can be obtained by frequency weighted model reduction methods. To explain this idea clearly, we replace \( G - G_r \) with \( F_1(P, K) - F_1(P, K_r) \) in power series of \( K - K_r \), and neglect the terms of second order in \( K - K_r \), to obtain

\[
\Delta = W_2^{-1}(F_1(P, K) - F_1(P, K_r))W_1^{-1}
\approx W_2^{-1}(P_{12}K(I - P_{22}K_r)^{-1}P_{21} - P_{12}K_r(I - P_{22}K^{-1}P_{21})W_1^{-1}
= W_2^{-1}P_{12}(I + K - P_{22}K_r)^{-1}P_{21}W_1^{-1}
= W_2^{-1}V_1(K - K_r)(I - P_{22}K_r)^{-1}P_{21}W_1^{-1}
= W_2^{-1}V_2(K - K_r)V_1W_1^{-1}
\]

where \( V_1 = (I - P_{22}K_r)^{-1}P_{21}, V_2 = P_{12}(I + K - P_{22}K_r)^{-1}P_{21} \).

The above formula clearly shows that \( G_r \) or \( K_r \) can be obtained by frequency weighted model reduction methods. To obtain a performance preserving low-order controller \( K_r \) by frequency weighted model reduction, we must choose the weightings \( W_1 \) and \( W_2 \) properly. In the following, we first investigate the condition for \( K_r \) to be \( (P, 1) \)-admissible when \( K \) is \( (P, 1) \)-admissible and then derive the formulas for \( W_1 \) and \( W_2 \).

Theorem III.1: Assume \( ||G_r||_\infty < 1 \). \( W_1 \) and \( W_2 \) are stable, invertible weighting functions with minimum phase and also satisfy

\[
\begin{bmatrix}
I - W_2W_2^{-1} & G \\
G^{-1} & I - W_1^{-1}W_1
\end{bmatrix} \geq 0 \quad \text{and} \quad ||W_2^{-1}(G - G_r)W_1^{-1}||_\infty < 1.
\]

Then

\[ ||G_r||_\infty < 1. \]

The proof of the above theorem requires the following lemma.

Lemma III.1: [1] Suppose a Hermitian matrix is partitioned as

\[
\begin{bmatrix}
A & B \\
B^* & C
\end{bmatrix}
\]

where \( A = A^* \) and \( C = C^* \). This matrix is positive-semidefinite if and only if \( A - BC^*B^* \geq 0 \).

Proof of Theorem III.1:

\[
\begin{align*}
||W_2^{-1}(G - G_r)W_1^{-1}||_\infty < 1 & \quad \iff \quad W_2^{-1}(G - G_r)W_1^{-1}(W_1^{-1})^* (G^* - G_r^*) \geq I \\
& \quad \iff \quad (G - G_r)(W_1^{-1}W_1)^{-1}(G^* - G_r^*) < W_2W_2^{-1} \\
& \quad \iff \quad W_2W_2^{-1} - G_r^* - G_r < 0 \\
& \quad \iff \quad 0 < G_r^* - G_r < W_2W_2^{-1} \\
& \quad \iff \quad I < W_2W_2^{-1} < W_1^{-1}W_1 \quad \text{and} \quad 0 < G_r^* - G_r < W_2W_2^{-1} \quad \iff \quad ||G_r||_\infty < 1.
\end{align*}
\]
Using the series expansion \((I + M\Psi)^{-1} = I - M\Psi + o(\Psi)\), one can derive
\[
f(Y + \Psi) = \text{trace} \left[ (X - (I - Y)^{-1}Y)^{-1}Z + o(\Psi) \right]
\times \left[ Y^{-1} - Y^{-1} \Psi Y^{-1} + o(\Psi) \right]
\]
where \(Z = [X - (I - Y)^{-1}Y(I - Y)^{-1}][X - (I - Y)^{-1}]^{-1}\). Therefore, \(Y\) is a stationary point if
\[
\text{trace} \left[ Y^{-1} - Y^{-1} \Psi Y^{-1} \right] = 0
\]
for any \(\Psi = \Psi^\infty\). It is routine to show that
\[
\min_{\Psi_1, \Psi_2} \sigma_1(W_2^{-1}GW_1^{-1}) = (x_i^{1/2} - 1)^{-1} = \left(\frac{g_i}{1 - g_i}\right)^{-1}
\]

**Proof:** As in Theorem III.2, we assume \((GG^\infty)^{-1}\) exists. \(X\) denotes \((GG^\infty)^{-1}\) and \(Y\) denotes \(W_2^\infty\).

Let \(g_i = \sigma_i(G)\), then
\[
\min_{W_1, W_2} \sigma_1(W_2^{-1}GW_1^{-1}) = \frac{g_i}{1 - g_i}, \quad \text{for all } i.
\]

**Remark III.2:** In the proof of Theorems III.2 and III.3, we assumed that \((GG^\infty)^{-1}\) exists. Instead of assuming \((GG^\infty)^{-1}\) exists, we prove these theorems alternatively, by assuming \((G^G)^{-1}\) exists. Note that if \(G\) is tall, then \((GG^\infty)^{-1}\) is positive-semidefinite and \((GG^\infty)^{-1}\) does not exist. Similarly, if \(G\) is long, \((G^G)^{-1}\) is positive-semidefinite and \((G^G)^{-1}\) does not exist. However, we only need either \((G^G)^{-1}\) or \((GG^\infty)^{-1}\) to exist, which is usually satisfied in practice. However, if this fails, we could always perturb \(G\) by adding \(\epsilon I\) and then letting \(\epsilon\) approach zero.

**Lemma III.2:** Assume that \(\|G\|_{\infty} < 1\), \(GG^\infty\) and \(W_2^\infty\) can be diagonalized simultaneously and
\[
\begin{bmatrix}
I - W_2GW_1^{-1} & G \\
G & I - W_1GW_1^{-1}
\end{bmatrix} \geq 0.
\]

Therefore
\[
\min_{W_1, W_2} \sigma_1(W_2^{-1}GW_1^{-1}) = (x_i^{1/2} - 1)^{-1} = \left(\frac{g_i}{1 - g_i}\right)^{-1}.
\]

**Remark III.2:** In the proof of Theorems III.2 and III.3, we assumed that \((GG^\infty)^{-1}\) exists. Instead of assuming \((GG^\infty)^{-1}\) exists, we prove these theorems alternatively, by assuming \((G^G)^{-1}\) exists. Note that if \(G\) is tall, then \((GG^\infty)^{-1}\) is positive-semidefinite and \((GG^\infty)^{-1}\) does not exist. Similarly, if \(G\) is long, \((G^G)^{-1}\) is positive-semidefinite and \((G^G)^{-1}\) does not exist. However, we only need either \((G^G)^{-1}\) or \((GG^\infty)^{-1}\) to exist, which is usually satisfied in practice. However, if this fails, we could always perturb \(G\) by adding \(\epsilon I\) and then letting \(\epsilon\) approach zero.

**Lemma III.2:** Assume that \(\|G\|_{\infty} < 1\), \(W_2GW_1^{-1}\) and \(G^G\) can be diagonalized simultaneously and
\[
\begin{bmatrix}
I - W_2GW_1^{-1} & G \\
G & I - W_1GW_1^{-1}
\end{bmatrix} \geq 0.
\]

Then, \(W_2GW_1 = I - \sqrt{G^GG}\) and \(W_1GW_1 = I - \sqrt{G^GG}\) is a pair of weightings which minimize \(\sigma_i(W_2^{-1}GW_1^{-1})\).

This lemma is a direct result of the above theorem. It is obvious that \(W_2GW_1 = I - \sqrt{G^GG}\) implies \(W_2GW_1\) and \(G^G\) can be diagonalized simultaneously, and at the same time, their eigenvalues satisfy (7).

**Lemma III.3:** If \(\|G\|_{\infty} < 1\), \(W_2GW_1\) and \(G^G\) can be diagonalized simultaneously and
\[
\begin{bmatrix}
I - W_2GW_1^{-1} & G \\
G & I - W_1GW_1^{-1}
\end{bmatrix} \geq 0.
\]

Therefore
\[
\min_{W_1, W_2} \sigma_1(W_2^{-1}GW_1^{-1}) = (x_i^{1/2} - 1)^{-1} = \left(\frac{g_i}{1 - g_i}\right)^{-1}.
\]

**Proof:** As in Theorem III.2, we assume \((GG^\infty)^{-1}\) exists. \(X\) denotes \((GG^\infty)^{-1}\) and \(Y\) denotes \(W_2^\infty\).

Let \(g_i = \sigma_i(G)\), then
\[
\min_{W_1, W_2} \sigma_1(W_2^{-1}GW_1^{-1}) = \frac{g_i}{1 - g_i}, \quad \text{for all } i.
\]
Lemma III.4: If \( \|G\|_\infty = \sigma < 1 \), \( W_2 \) is in form of \( \alpha I \), where \( \alpha \) is a constant, and
\[
\begin{bmatrix}
I - W_2 W_2^- G \\
G \\
G W_2^- W_1 \\
I - W_1^- W_1
\end{bmatrix} \geq 0
\]
then \( W_2 = (1 - \sigma)^{1/2} I \) and \( W_2^- W_1 = I - \sigma^{-1} G^{-1} G \) are a pair of weightings that minimize \( \sigma_r(W_2^{-1} G W_1) \).

The above pair of weightings is optimal when \( W_2 \) is in the form of \( \alpha I \) and is very much easier to compute. Another advantage of this pair is that the order of whole frequency weighted system will be reduced because \( W_2 \) can be removed from the reduction procedure. The performance of this pair of weightings can be seen from the examples in the next section.

Finally, we consider the situation when \( W_2^- W_1 \) and \( W_2 W_2^- \) are both in form of \( \alpha I \), where \( \alpha \) is a constant. It is easy to prove that \( W_2^- W_1 = (1 - \alpha) I \) and \( W_2 W_2^- = (1 - \sigma) I \) or \( W_2^- W_1 = (1 - \alpha) I \) and \( W_2 W_2^- = (1 - \sigma) I \) are the best choices.

In this case
\[
\|W_2^{-1}(G - G_r)W_1^-\|_\infty = \|(1 - \alpha) I - (G - G_r)(1 - \sigma) I^{-1/2}\|_\infty = (1 - \sigma)^{-1/2}\|G - G_r\|_\infty.
\]
This means minimizing \( \|W_2^{-1}(G - G_r)W_1^-\|_\infty \) is equivalent to minimizing \( \|G - G_r\|_\infty \) and therefore these weightings seem useless.

For all three pairs of weightings, it is easy to show that
\[
\|W_2^{-1}G W_1^-\|_\infty = \frac{\sigma}{1 - \sigma}.
\]
Therefore, when designing frequency weightings, it is necessary not only to minimize \( \|W_2^{-1} G W_1\|_\infty \) but also to minimize \( \sigma_r(W_2^{-1} G W_1) \). The difference between the controller order reduction methods based on minimizing \( \|G - G_r\|_\infty \) and performance preserving methods is the choice of weightings. The controller reduction methods based on minimizing \( \|G - G_r\|_\infty \) are special cases of performance preserving controller reduction methods when weightings are constants.

Controller Order Reduction Procedure:

Step 1) Scaling: Suppose the transfer matrix \( G \) of the closed-loop system with full-order controller has been scaled such that \( \|G\|_\infty = \sigma < 1 \).

Step 2) Compute Weights: Compute \( W_1 \) and \( W_2 \) using the formulas in Theorem III.2 or Lemma III.4.

Step 3) Compute Low-Order Controller: Two different methods can be employed here. First, solve \( \|W_2^{-1}(G - G_r)W_1^-\|_\infty \) reduction directly to obtain \( K_r \). Methods to solve \( \|W_2^{-1}(G - G_r)W_1^-\|_\infty \) reduction directly are given in [12]. Second, using (4) and (5), approximate \( \|W_2^{-1}(G - G_r)W_1^-\|_\infty \) reduction by \( \|W_2^{-1}V_2(K - K_r)V_1 W_1^-\|_\infty \) reduction and then use frequency weighted model reduction methods to obtain \( K_r \).

Remark III.3: In practice, in order to know if \( K_r \) is a \( (P, 1) \)-admissible controller, it is not necessary to test \( \|W_2^{-1}(G - G_r)W_1^-\|_\infty < 1 \), because Theorem III.1 is only a sufficient condition. If \( \|W_2^{-1}(G - G_r)W_1^-\|_\infty \geq 1 \) it does not mean \( K_r \) is not a \( (P, 1) \)-admissible controller. Furthermore, if there are some errors in computing \( W_1 \) and \( W_2 \) (which can not be prevented in rational approximation), even \( \|W_2^{-1}(G - G_r)W_1^-\|_\infty < 1 \) can not guarantee that \( K_r \) is a \( (P, 1) \)-admissible controller. Finally, the computation of \( \|W_2^{-1}(G - G_r)W_1^-\|_\infty \) is not easier than the computation of \( \|G_r\|_\infty \). Therefore, in order to know if \( K_r \) is a \( (P, 1) \)-admissible controller, a simple and reliable way is to compute \( \|G_r\|_\infty \) directly.

IV. Computational Issues and Examples

In the last section, we derived two pairs of weightings. When \( W_2^- W_1 = I - \sigma^{-1} G^{-1} G \), we can obtain an exact solution to \( W_1 \) by spectral factorization. When \( W_2^- W_1 = I - \sqrt{G^{-1} G} \) we need to compute a rational approximation for \( \sqrt{G^{-1} G} \) first. Generally, we can obtain a rational function that is arbitrarily close to \( \sqrt{G^{-1} G} \) using appropriate rational approximation algorithms [2]. Usually, the accuracy of the rational approximation increases the order of \( W_1 \) and \( W_2 \). Therefore, we need to make a tradeoff between the order of \( W_1 \) and \( W_2 \) and the accuracy of the approximation. Compared with Goddard and Glover’s method, the order of weightings in our method is very low. Given a plant of order \( n \), in Goddard and Glover’s method, the order of \( K \) will be \( n \) and the order of \( L \) will be \( 4n \). The order of \( \tilde{Z}_1 \) and \( \tilde{Z}_2 \) may be up to \( 12n \), hence the first order rational approximation \( \tilde{Z}_1 \) and \( \tilde{Z}_2 \) will be \( 12n \). Therefore, the order of the first order rational approximation of \( W_2^- W_1 \) or \( W_2 W_2^- \) in Goddard and Glover’s method is \( 12n \) at least. However, in our method the order of the first order rational approximation of \( W_2^- W_1 \) or \( W_2 W_2^- \) is only \( 4n \). Once \( W_1 \) and \( W_2 \) are obtained, the controller order reduction problem \( \|W_2^{-1}(G - G_r)W_1^-\|_\infty \) reduction problem can be solved by the following two methods.

1) By approximating \( \|W_2^{-1}(G - G_r)W_1^-\|_\infty \) reduction by \( \|W_2^{-1}V_2(K - K_r)V_1 W_1^-\|_\infty \) and then solving the frequency weighted model reduction problem, the reduced-order controller is obtained. Note that from (2)–(6), minimizing \( \|W_2^{-1}V_2(K - K_r)V_1 W_1^-\|_\infty \) is approximately equal to minimizing \( \|W_2^{-1}(G - G_r)W_1^-\|_\infty \). Therefore, we can use frequency weighted model reduction problem or \( \|W_2^{-1}V_2(K - K_r)V_1 W_1^-\|_\infty \) reduction to solve performance preserving controller reduction problem. Note that although this procedure introduces new weights \( V_1 \) and \( V_2 \), the combined order of \( W_1 \) and \( V_2 \), and \( V_2 \) is only \( 8n \) which is still less than \( 12n \), the order of weights obtained using Goddard and Glover’s method.

2) The problem can also be solved by directly solving \( \|W_2^{-1}(G - G_r)W_1^-\|_\infty \) reduction problem. Note that in both the methods \( K_r \) is obtained directly. There is no additional overhead involved in obtaining \( K_r \) using the second method. Please see [12] for more details. Note that the difference between \( \|W_2^{-1}(G - G_r)W_1^-\|_\infty \) reduction problem solved in [12] and \( \|G - G_r\|_\infty \) reduction problem solved in [7], [9] is the weightings. If a method is suitable to solve \( \|G - G_r\|_\infty \) reduction problem, the method can be easily used to solve \( \|W_2^{-1}(G - G_r)W_1^-\|_\infty \) reduction problem.

Remark IV.1: The weights in Goddard and Glover’s method are derived using the following two measures: 1) minimizing the product \( \text{trace}(W_2^- W_1) \) and minimizing the product \( \text{det}(W_2^- W_1) \). The weights in our method are derived by minimizing: 1) \( \text{trace}(W_2^{-1} G W_1^{-1} G^{-1}) \), and 2) \( \sigma_r(W_2^{-1} G W_1^{-1} G^{-1}) \). Although, the methods employed to determine the weights are different, the motivation behind them are the same: to make the weights \( W_1 \) and \( W_2 \) large in some sense to make the subsequent approximation as easy possible.

Example: We consider the design of reduced-order robust controllers for the longitudinal dynamics of an experimental highly maneuverable (HIMAT) airplane [11]. In the example, the generalized plant \( P \) has 20 states. \( K \) is a 20-state controller such that \( \|G\|_\infty = 0.974 \). Using the weighting formula proposed in Lemma
TABLE I
THE RESULTS OF CONTROLLER REDUCTION

<table>
<thead>
<tr>
<th>order of $K$</th>
<th>19</th>
<th>18</th>
<th>17</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>$| F(P, K) |_\infty$</td>
<td>0.974</td>
<td>0.974</td>
<td>0.974</td>
<td>0.974</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>order of $K$</th>
<th>15</th>
<th>14</th>
<th>13</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$| F(P, K) |_\infty$</td>
<td>0.974</td>
<td>0.976</td>
<td>0.984</td>
<td>0.983</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>order of $K$</th>
<th>11</th>
<th>10</th>
<th>7</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$| F(P, K) |_\infty$</td>
<td>0.993</td>
<td>1.209</td>
<td>1.22</td>
<td>1.22</td>
</tr>
</tbody>
</table>

TABLE II
THE RESULTS FROM GODDARD AND GLOVER’S PAPER

<table>
<thead>
<tr>
<th>Reduction Approach</th>
<th>controller Order</th>
<th>$| WE |_\infty$</th>
<th>$| F(P, K) |_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A($\Phi$)</td>
<td>14</td>
<td>0.67</td>
<td>0.99</td>
</tr>
<tr>
<td>CF($\Phi$)</td>
<td>16</td>
<td>0.94</td>
<td>0.98</td>
</tr>
<tr>
<td>CF($\Theta$)</td>
<td>16</td>
<td>0.03</td>
<td>0.99</td>
</tr>
<tr>
<td>UOHN A</td>
<td>13</td>
<td>-</td>
<td>0.98</td>
</tr>
<tr>
<td>UBT</td>
<td>13</td>
<td>-</td>
<td>0.98</td>
</tr>
<tr>
<td>LCFR</td>
<td>13</td>
<td>-</td>
<td>0.98</td>
</tr>
<tr>
<td>A($\Phi$)</td>
<td>7</td>
<td>11.27</td>
<td>1.27</td>
</tr>
<tr>
<td>CF($\Phi$)</td>
<td>7</td>
<td>956</td>
<td>7.07</td>
</tr>
<tr>
<td>CF($\Theta$)</td>
<td>10</td>
<td>12.44</td>
<td>2.02</td>
</tr>
</tbody>
</table>

III.4 and PPB reduction method to reduce the order of $K$, we get the following results.

For comparing with Goddard and Glover’s method [11], we have reprinted the table from their paper [11] as follows.

In Table I, $WE$ denotes weighted error and A($\Phi$), CF($\Phi$) and CF($\Theta$) are the methods proposed in [11]. UOHN A represents unweighted optimal Hankel norm approximation, UBT represents unweighted balanced truncation, and LCFR represents unweighted optimal Hankel norm reduction of controller left coprime factors. From Table II, we can see that the lowest order of performance preserving controller using the proposed method is 11 whereas using Goddard and Glover’s method [11] it is 14 (without optimization).

V. CONCLUSION

Given a $(P, 1)$-admissible controller, $K$, sufficient conditions for a low-order controller, $K_0$, to be $(P, 1)$-admissible are derived. The conditions are given in terms of 1) positive definiteness of some matrix involving weights and the closed-loop transfer function, and 2) a norm bound on a particular frequency weighted error. If these conditions are satisfied, then it is possible to reduce the order of $H_\infty$ controllers without degrading performance. Two approaches presented require calculation of weights. Compared with the method of [11], the order of weights required here are much lower which can reduce computational complexity involved. As in Goddard and Glover’s method, the first set of weights presented requires the computation of nonstandard transfer function factorization which can not be performed exactly. However, the second of set of weights presented are easier to compute and as shown in the numerical example produces good results. Therefore, the method can be used as an attractive alternative to Goddard and Glover’s method. Comprehensive comparing of the two methods needs further research.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their useful comments, and P. Goddard for supplying the data for the numerical example.

REFERENCES