1. The linearized equations of a free pendulum are $\ddot{\theta} + \omega_0^2 \theta = u$. Show that output feedback $-k\theta$ will not stabilize the system but that this can be done by using a system $(s + \alpha)(s + \beta)$ in the feedback path, provided $\alpha < \beta$.

2. Consider a 2nd order system described the following state space equations:

$$\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
-1 & 0 \\
0 & 2
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
0 \\
1
\end{bmatrix} u$$

$$y = \begin{bmatrix}
c_1 & c_2
\end{bmatrix}$$

a) Is the system controllable?

b) Is the system observable?

c) Find a feedback gain vector, $k$ to place the system poles at \{-1, -2\}, if possible.

d) Repeat c) to place poles at \{-2, -2\}.

e) Explain any discrepancy in answers to (c) and (d).

f) Which state variable should be measured to stabilize the system using feedback?

g) Find the transfer function when $c_1 = 1$ and $c_2 = 0$.

h) Repeat g) for $c_1 = 0$ and $c_2 = 1$.

3. A helicopter near hover can be described by the equations

$$\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} = \begin{bmatrix}
-0.02 & -1.4 & 9.8 \\
-0.01 & -0.4 & 0.0 \\
0.0 & 1.0 & 0.0
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} + \begin{bmatrix}
9.8 \\
6.3 \\
0.0
\end{bmatrix} u$$

where $x_1$ = horizontal velocity, $x_2$ = pitch rate, $x_3$ = pitch angle, $u$ = rotor tilt angle. (a) Find the open loop poles (b) Show that a state-feedback law to move the poles to $s = -2$, $s = -1 \pm j$ is $k = [0.0628, \ 0.4706, \ 0.9949]$
4. Consider the system defined by

\[ \dot{x} = Ax + Bu \]

where

\[ A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \]

By using the state feedback control \( u = -Kx \), it is desired to have the closed-loop poles at \( s = -2 \pm j4, \ s = -10 \). Determine the state feedback gain matrix \( K \).