Solutions to Examples

Consider the polynomial: \( A_1(s) = s^4 + 3s^3 - 14s^2 - 48s - 32 \)

Routh table:

\[
\begin{array}{cccc}
  s^4 & 1 & -14 & -32 \\
  s^3 & 3 & -48 \\
  s^2 & 2 & -32 \\
  s^1 & 0 & \rightarrow \epsilon \\
  s^0 & -32 \\
\end{array}
\]

Since there is one sign change in the first column, there is one unstable root (or one root in the right half of s-plane).
Consider the polynomial: \[A_2(s) = s^5 + 2s^4 - 17s^3 - 34s^2 + 16s + 32\]

Routh table:

\[
\begin{array}{cccc}
 s^5 & 1 & -17 & 16 \\
 s^4 & 2 & -34 & 32 \leftarrow \text{Auxiliary Polynomial } P(s) \\
 s^3 & 0 & 0 \leftarrow \text{Row of zeros} \\
 s^3 & 8 & -68 \leftarrow \text{Replace with coefficients of } \frac{dP(s)}{ds} \\
 s^2 & -17 & 32 \\
 s^1 & -52.94 \\
 s^0 & 32 \\
\end{array}
\]

Auxiliary polynomial: \[P(s) = 2s^4 - 34s^2 + 32, \text{ and } \frac{dP(s)}{ds} = 8s^3 - 68s\]

Since there are 2 sign changes in the first column, there are 2 unstable roots.
Consider the polynomial:

\[ A_3(s) = s^3 + 3s^2 + s + 3 \]

Routh Array:

\[
\begin{array}{ccc}
  s^3 & 1 & 1 \\
  s^2 & 3 & 3 \\
  s^1 & 0 \rightarrow \varepsilon \\
  s^0 & 3 \\
\end{array}
\]

There are no sign changes in the first column of the Routh array, and therefore there are no roots in the right half of s-plane. However, there are two roots on the boundary (jw-axis). The roots on jw-axis can be obtained by calculating the roots of the auxiliary polynomial:

\[ P(s) = 3s^2 + 3, \text{ roots of } P(s) = \pm j \]
Consider the polynomial: \[ A_4(s) = s^4 + 3s^3 + 18s^2 + 48s + 32 \]

Routh array:

\[
\begin{array}{cccc}
  s^4 & 1 & 18 & 32 \\
  s^3 & 3 & 48 & \\
  s^2 & 2 & 32 & \\
  s^1 & 0 & \rightarrow & \epsilon \\
  s^0 & 32 & \\
\end{array}
\]

There are no sign changes in the first column of the Routh array. Therefore there are no roots in the right half of s-plane. However, there are roots on the \( jw \)-axis due to a zero in the first column. The roots on the \( jw \)-axis can be easily calculated by constructing the auxiliary polynomial using the coefficients in the row above the zero.

\[ P(s) = 2s^2 + 32, \text{ roots of } P(s) = \pm j4 \]
Consider the polynomial $A_5(s) = s^5 + 8s^4 + 25s^3 + 40s^2 + 34s + 12$

Routh table:

\[
\begin{array}{ccc}
  & s^5 & 1 & 25 & 34 \\
  & s^4 & 8 & 40 & 12 \\
  & s^3 & 20 & 32.5 \\
  & s^2 & 27 & 12 \\
  & s^1 & 23.61 \\
  & s^0 & 12 \\
\end{array}
\]

Since there are no sign changes in the first column of the Routh array, there are no unstable roots. All roots are in the left half of s-plane.
Consider the closed-loop system:

\[
\frac{C(s)}{R(s)} = \frac{K}{s^4 + 3s^3 + 3s^2 + 2s + K}
\]

For stability we want:

\(K > 0, \text{ and } \frac{14}{3} - 3K > 0\)

or \(0 < K < \frac{14}{9}\)

Routh Array

\[
\begin{array}{ccc}
\ s^4 & 1 & 3 & K \\
\ s^3 & 3 & 2 & \\
\ s^2 & \frac{7}{3} & K & \\
\ s^1 & \frac{(14/3) - 3K}{7/3} & \\
\ s^0 & K & \\
\end{array}
\]