1. (a) By looking at the realization shown by the signal flow graph, comment on the controllability and observability and give reasons (2 marks).

(b) Write the state-space equations for the realization shown in the signal flow graph (2 marks).

(c) Determine the controllability and observability of the realization using controllability and observability matrices (2 marks).

1 (a) The realization is completely controllable because all states are influenced by the input either directly or indirectly. The realization is completely observable because the output is influenced by all the states either directly or indirectly.

1 (b)

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
-4 & -3 & -2
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} u \\
y = \begin{bmatrix}
0 & 5 & 1
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
\]
1 (c) Controllability matrix is given by

\[ C = \begin{bmatrix} b & Ab & A^2b \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & X \\ 1 & X & X \end{bmatrix}, \quad |C| = -1 \neq 0, \quad \therefore \{A, b\} \text{ is controllable.} \]

Observability matrix is given by

\[ O = \begin{bmatrix} c \\ cA \\ cA^2 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 1 \\ -4 & -3 & 3 \\ -12 & -13 & -9 \end{bmatrix}, \quad |O| = -344 \neq 0, \quad \therefore \{c, A\} \text{ is observable.} \]

Therefore, the realization is both controllable and observable.
2. Determine the pulse-transfer function $C(z)/R(z)$ for the sampled-data system shown in the figure: (4 marks)

\[
C(s) = G_1(s)G_2(s)E^*(s) = G_1G_2(s)E^*(s)
\]

\[
E(s) = R(s) - C(s)H(s) = R(s) - G_1(s)G_2(s)H(s)E^*(s) = R(s) - G_1G_2H(s)E^*(s)
\]

\[
E^*(s) = R^*(s) - G_1G_2H^*(s)E^*(s)
\]

\[
E^*(s) = \frac{1}{R^*(s) + G_1G_2H^*(s)}
\]

\[
C^*(s) = G_1G_2^*(s)E^*(s) = \frac{G_1G_2^*(s)R^*(s)}{1 + G_1G_2H^*(s)}
\]

\[
\frac{C^*(s)}{R^*(s)} = \frac{G_1G_2^*(s)}{1 + G_1G_2H^*(s)}, \text{ or } \frac{C(z)}{R(z)} = \frac{G_1G_2(z)}{1 + G_1G_2H(z)}
\]

where $G_1G_2(z) = Z\{G_1(s)G_2(s)\}$ and $G_1G_2H(z) = Z\{G_1(s)G_2(s)H(s)\}$