The Pigeon-Hole Principle: Problems

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1. In a group of 8 people show that at least two have their birthday on the same day of the week.

2. Three numbers are chosen at random. Their sum is 19. Show at least one number is 7 or more.

3. A box contains 10 French books, 20 Spanish books, 8 German books, 15 Russian books, and 25 Italian books. How many must we choose to ensure that we have 12 books in the same language?

4. There are 30 students in a class. While doing a keyboarding test one student made 12 mistakes, while the rest made fewer mistakes. Show that at least 3 students made the same number of mistakes.

5. A teacher starts each year with 3 jokes. Over 12 years the teacher never repeated the same triple of jokes. What is the smallest number of jokes the teacher must have in her repertoire for this to be possible?

6. A canteen has 95 tables with a total of 465 seats. Can we be sure that there is a table with at least 6 seats?

7. Prove that of any 5 points chosen in an equilateral triangle of side-length 1, there are two points whose distance apart is at most $\frac{1}{2}$.

8. Suppose we have 27 distinct positive odd numbers ... all less than 100. Show there is a pair of numbers whose sum is 102.

9. The numbers 1 to 10 are arranged in random order around a circle. Show that there are three consecutive numbers whose sum is at least 17.

10. Six swimmers training together either swam in a race or watched the others swim. At least how many races must have been scheduled if every swimmer had opportunity to watch all of the others?

11. There are 15 people at a party. Some of them exchange handshakes with some of the others. Prove that at least two people have shaken the same number of hands.

12. A computer is used for 99 hours over a period of 12 days. Prove that on some pair of consecutive days the computer was used at least 17 hours.
13. Show that given any 17 natural numbers it is possible to choose 5 whose sum is divisible by 5.

14. A circle is divided into 8 equal sectors. Half are coloured red and half are coloured blue. A smaller circle is also divided into equal sectors, half coloured red and half coloured blue. The smaller circle is placed concentrically on the larger. Prove that no matter how the red and blue sectors are chosen it is always possible to rotate the smaller circle so that at least 4 colour matches are obtained. (The diagram below shows an example.)

15. Five microcomputers are to be connected to three printers. How many connections are necessary between computers and printers in order to ensure that whenever any three computers require a printer the printers are available?

16. Prove that, of any 5 points chosen within a square of side-length 2, there are two whose distance apart is at most $\sqrt{2}$.

17. A disk of radius 1 is completely covered by 7 identical smaller disks. (They may overlap.) Show that the radius of each of the smaller disks must not be less than $\frac{1}{2}$.

18. A graph consists of vertices (singular: vertex) and edges. Vertices are usually represented by filled-in dots and each edge starts and finishes at a vertex. The degree of a vertex is the number of edges that start (or finish) at that vertex. Suppose a graph has 9 vertices such that each vertex has degree 5 or 6. Prove that at least 5 vertices have degree 6 or at least 6 vertices have degree 5.

19. How many trees can farmer Fred plant on his 100 m square field if they are to be no closer than 10 m apart? (Neglect the thickness of the trees.)