Set Theory, Logic and Boolean Algebra: Problems

We say below that we try to find regions that cover 2-powers of $\times$s. The shapes of these regions can’t be arbitrary; they must be “regular” such that they correspond to intersections of variables, e.g. for three variables $A, B, C$, $ABC$ covers 1 square, $AB$ covers 2 squares, and $A$ covers 4 squares. In practice it is easy to recognise which “2-power regions” are the appropriate ones. Note that engineers usually write the complement of $A$ as $\overline{A}$ rather than $A'$; and we’ll follow that convention below.

1. Simplify the Boolean expression $A\overline{B} + A(B + C) + B(B + \overline{C})$

2. Simplify $(A\overline{B}(C + BD) + A\overline{B})C$

3. Simplify $\overline{A}BC + A\overline{B}C + \overline{A}\overline{B}C + A\overline{B}C + ABC$

4. Simplify $\overline{B}\overline{C} + A\overline{B} + AB\overline{C} + ABC\overline{D} + \overline{A}\overline{B}\overline{C}D + A\overline{B}CD$

5. Let $A, B, C, D$ represent the binary digits of a decimal number in the range 0 to 15. Construct a simplified expression that returns 1 exactly when $(ABCD)_{\text{two}}$ is the binary representation of an odd composite number.

6. Using the fact that $(p \rightarrow q) \iff (\neg p \lor q)$, find a LHS $\iff \cdots \iff$ RHS proof of

   $$\neg q \rightarrow \neg p \iff (p \rightarrow q).$$

   \textit{Note.} $\neg q \rightarrow \neg p$ is the contrapositive of $p \rightarrow q$. You will have proved that these statements are equivalent. In the note, this result is proved by truth table.