Deterministic nonlinearity in ventricular fibrillation

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We provide numerical evidence that the electrocardiogram data collected from pigs during induced ventricular fibrillation cannot be described by a monotonic nonlinear transformation of linearly filtered noise. To establish this we use surrogate techniques and apply two test statistics: (1) the Takens’ maximum likelihood estimator of the Grassberger–Procaccia correlation dimension and (2) an improved correlation dimension estimation routine. The improved dimension estimates provide evidence that the correlation dimension of the underlying dynamics during the episode of VF in the first 30 s is slightly less than 6. This result is consistent and reproducible among subjects. © 2000 American Institute of Physics. [S1054-1500(00)00601-7]

I. INTRODUCTION

Cardiovascular disease is the leading cause of death in the industrial world. In the USA alone, it causes more than 300 000 sudden deaths each year. Ventricular fibrillation (VF) is the most common mode of death among patients with this condition. During VF, the irregular electrical wave activity destroys the coherent contraction of the ventricular muscle and its main pumping function. The heart rate is too high (>550 excitations/min) to allow any antegrade pumping of blood. As a result, there is no cardiac output, peripheral pulses and blood pressure are absent, and the body, in particular the brain, is deprived of oxygen and the patient loses consciousness. Death follows within minutes unless the arrhythmia is immediately controlled.

VF was first described in 1849.1 Today it is routinely treated with electrical defibrillation with a large (typically 200–360 J) DC electric shock. Despite this long history and well-established treatment regime the underlying cause of VF is poorly understood. Theoretical and computational work has demonstrated that VF may be attributed to a complex nonlinear pattern formation of drifting spiral waves of electrical activity (vortices or rotors) across the myocardium and their subsequent breakdown.2–7 It is therefore important to establish whether electrical activity observed via an electrocardiogram (ECG) during VF is nonlinear in its dynamical behavior (and possibly chaotic) or it can simply be modeled as a linear stochastic process. In this paper we present experimental evidence that VF cannot be described as a static monotonic nonlinear transformation of a linearly filtered noise source. A mathematical description of VF must involve either dynamic nonlinearity or a static nonmonotonic nonlinear transformation.

Ventricular fibrillation (VF) is an irregular, disorganized electrical activity of the heart during which the normal electrical patterns can no longer be identified. The recorded deflection continuously changes in shape, duration, magnitude and direction. Alternative theories of the origin of VF hold that it behaves either as correlated noise or as a low-dimensional deterministic nonlinear motion. In this paper we present experimental evidence that such a seemingly random electrical activity cannot be described by a linear noise model. We apply two nonlinear techniques to estimate the correlation dimension and compare these estimates to those from different realizations of linear noise models (linear surrogates). Previous authors have estimated correlation dimensions for VF data. However all published results have used the flawed Grassberger–Procaccia algorithm (or variants on this) and few applied surrogate data techniques. Here we apply an alternative dimension estimation routine, Judd’s estimate, which has been shown to be reliable and robust even for short noisy time series. The Takens’ maximum likelihood estimate of the Grassberger–Procaccia algorithm is used to calculate the significance between raw and surrogate data in testing for nonlinearity. Our results demonstrate that correlation dimension estimates of VF data are distinct from those expected for noise driven linear systems. Furthermore, our calculations show that VF exhibits low dimensionality with a correlation dimension between five and six. Our results are consistent for different time series recorded from different subjects, and for different dimension estimation routines.

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A. VF—Chaos or noise?

Nonlinear dynamics and deterministic chaos offer an attractive alternative to the thesis that VF is the random evolution of electrical activity on the myocardium. Many authors have studied VF data collected in a variety of ways from different sources in an attempt to determine whether it is deterministic chaos or stochastic noise. The conclusions have been mixed, sometimes contradictory.

Several authors have analyzed data from heart tissue and other biologically stationary media or artificial signals. Much attention has also been focused on the beat to beat dynamics contained in sequences of RR intervals (e.g., Ref. 8). In this paper we examine the complete ECG wave forms observed from functioning pig hearts.

Goldberger et al.10 examined estimates of the power spectrum of data recorded during VF in dogs. They observed a narrow band peak in the power spectrum which is indicative of noisy periodic behavior and not deterministic chaos. However, their calculations were based on relatively short segments of data (1 s recorded at 512 Hz) digitized by hand from strip charts.

Kaplan and Cohen11 also collected data from dogs during VF. They studied this data by applying a time delay embedding and estimating correlation dimension. These calculations failed to produce convergent correlation dimension estimates for the embedded data. They concluded that ventricular fibrillation is stochastic—not deterministic.

Ravelli and Antolini12 examined human ECG data in four distinct states: sinus rhythm, sinus rhythm with irregular beats (ventricular extrasystoles), coarse (early) VF, and fine (late) VF. Dimension estimation calculations using the algorithm of Grassberger and Procaccia13,14 demonstrated that sinus rhythm behaves as a low dimensional motion while the correlation dimension during VF is higher. Calculations of the correlation dimension of early VF gave estimates of about 5.7, late VF did not yield convergent estimates. They argued that early VF was consistent with a low dimensional chaotic dynamical system whereas late VF was not.

Finally, Govindan, Narayanan, and Gopinathan15 applied correlation dimension estimation and surrogate data techniques to various irregular ECG wave forms (premature ventricular contraction, ventricular tachyarrhythmia, atrio-ventricular block, and VF). They justly treated the “signatures of chaos”—correlation dimension and Lyapunov exponents estimated by various algorithms13,14,16—with some suspicion. However, they still applied these standard and somewhat dubious algorithms. To provide a safeguard against this they made limited use of surrogate data techniques and tested for variation in dimension estimates for different parameter values. They concluded that the ECG wave forms they observed were not consistent with linearly filtered noise. Positive Lyapunov exponents and correlation dimension estimates of 5.9 and 5.1 (for two time series of VF) provided them with further evidence of nonlinear determinism—and chaos. In this paper surrogate calculation results were stated for only one data set. In another work,17 the same authors demonstrated the (possible) existence of unstable periodic orbits in the same collection of irregular ECG wave forms. These unstable periodic orbits were indicative of nonlinearity and were consistent with chaotic deterministic behavior.

B. Distinguishing chaos from noise

The studies discussed in Sec. I A have all attempted to distinguish noise from chaos. However, this body of work has not offered an unambiguous statement. Specifically, while most evidence seems to be consistent with the hypothesis of low dimensional chaos, this does not mean however that the findings are inconsistent with linearly correlated noise.

In this paper we offer a new analysis of VF data which resolves the issue; we show that VF is not consistent with data generated by a monotonic nonlinear transformation of linearly filtered noise. We first apply extensive surrogate data tests and use a correlation dimension algorithm based on a maximum likelihood estimator of the Grassberger–Procaccia algorithm as a test statistic. Correlation dimension estimation algorithms are employed to provide a quantitative estimate of the degree of determinism (the number of active degrees of freedom) observed during VF. However, many algorithms do not provide estimates that are necessarily distinct from results one would obtain from linearly filtered noise. To overcome this we use a new and mathematically more sophisticated correlation dimension estimation algorithm, proposed by Judd, and apply the method of surrogate data. This algorithm has been favorably compared to existing techniques19 and has been successfully applied to experimental systems20,21 and shown to be robust against noise and small sample size.22 The linear techniques of surrogate data analysis are employed to provide a “sanity check” (to demonstrate that the correlation dimension estimates are distinct from results obtained for linear noise) and as a form of hypothesis test (to rigorously reject the hypothesis that the data are consistent with various linear systems). Surrogate data analysis requires the application of an (almost arbitrary) test statistic. We employ the above-mentioned correlation dimension algorithm to construct test statistics in our surrogate data analysis. In Sec. II we discuss each of these methods in more detail.

II. METHODOLOGY

In this section we describe the main mathematical techniques we employ in this paper. Section II A describes the correlation dimension estimation algorithms we used and Sec. II B describes the principle of surrogate data analysis.

A. Correlation dimension as a test statistic

Measures derived from nonlinear dynamical systems theory have proved to be extremely useful in many situations. However, the algorithms commonly employed to estimate both correlation dimension13,14 and Lyapunov exponents16 must be used with extreme care. Lyapunov exponents in particular are extremely difficult to estimate from experimental data (see, e.g., Ref. 23). When applied to finite noisy data the standard Grassberger–Procaccia algorithm...
Given a scalar time series, the most popular technique is the method of time delay embedding. Given a scalar time series \( \{ x_n \}_{n=1}^{N_t} \) of scalar observations, a vector time series is constructed via
\[
x_n \rightarrow \mathbf{v}_n := (x_n, x_{n-\tau}, x_{n-2\tau}, \ldots, x_{n-(d_e-1)\tau}),
\]
where \( \tau \) is the embedding lag, and \( d_e \) is the embedding dimension. Both these embedding parameters must be selected in some way. Various techniques to estimate these parameters have been suggested by many authors, for a review see Refs. 23 and 24.

The Grassberger–Procaccia method estimates correlation dimension by calculating the density of the attractor in phase space. The correlation integral \( C_{N,d_e}(\epsilon) \) is defined as
\[
C_{N,d_e}(\epsilon) = \lim_{N \to \infty} \left( \frac{N - W}{2} \right)^{-1} \sum_{|i-j| > W} H(\epsilon - \| \mathbf{v}_i - \mathbf{v}_j \|),
\]
where \( \mathbf{v}_i \) and \( \mathbf{v}_j \) are embedded points in the vector time series of length \( N = N_t - (d_e - 1)\tau \), \( \| \cdot \| \) is the usual Euclidean norm, and \( H(\cdot) \) is the Heaviside step function, defined by
\[
H(x) = \begin{cases} 
1 & \text{if } x > 0 \\
0 & \text{otherwise}.
\end{cases}
\]

In Eq. (2), the pairs of two points \((i,j)\) which are closer than some correlation distance \( W \) must be excluded. Grassberger and Procaccia\(^{13,14}\) assumed that the correlation dimension \( d_c \) is related to the correlation integral by
\[
C_{N,d_e}(\epsilon) \approx \epsilon^{d_c}.
\]
Further, Judd assumed that a fractal attractor can be modeled as the cross product of a Euclidean space and a “Cantor-like” set.\(^{18}\) Using this he showed that a closer approximation is that for all \( \epsilon < \epsilon_0 \),
\[
C_{N,d_e}(\epsilon) \approx \epsilon^{d_c} p(\epsilon),
\]
where \( p(\cdot) \) is a polynomial of degree at least 1 (in practice degree 1 is often sufficient). Because of the condition that \( \epsilon < \epsilon_0 \) this method produces a correlation dimension estimate \( d_c(\epsilon_0) \) which is a function of scale length \( \epsilon_0 \). Estimating correlation dimension as a function of scale has some useful advantages which are discussed in Refs. 22 and 20.

### B. Linear surrogate analysis

Surrogate analysis techniques\(^{25-27}\) provide definite algorithms by which one may test various hypotheses concerning an experimentally observed time series. From a time series one applies these algorithms to generate artificial data, surrogate data, which are “like” the raw data (conserve particular linear properties) but are also consistent with a particular hypothesis. By repeating this process many times and measuring a statistical quantity (test statistic) of the surrogate data one obtains a Monte Carlo estimate of the distribution of expected values of the test statistic for the time series data under the assumption that it is consistent with a given hypothesis. Comparing this probability distribution with the actual test statistic value for the data one may reject, or fail to reject the null hypothesis.

The three most commonly employed algorithms address the three hypotheses that the data is:\(^{25}\) (0) independent and identically distributed random noise, (1) linearly filtered noise, and (2) a monotonic nonlinear transformation of linearly filtered noise. The surrogates we generate in this paper are the so-called algorithm 2 surrogates. These surrogates address the hypothesis of monotonic nonlinear transformation of linearly filtered noise. Surrogates consistent with this hypothesis are generated by shuffling the data so that they have the same rank distribution as a colored noise time series with the same power spectrum as the original data. Details of this technique are described in more detail in the references discussed in the following paragraphs.

Concerns about the method used to generate algorithm 2 surrogates have been raised by some authors,\(^{28,29}\) however, for our data we did not find this to be a significant problem. Furthermore there are theoretical reasons for believing that correlation integral based test statistics are independent of these issues.\(^{30}\)

Recently there have been several attempts to generalize the method of surrogate data beyond these three algorithms. Theiler\(^{31}\) and Theiler and Rapp\(^{32}\) have proposed a useful algorithm to test for long term correlations in almost periodic time series. Schreiber and others have suggested a polished version of algorithm 2\(^{28}\) in which surrogates follow the desired spectrum more closely and a computational intensive generalization of the surrogate methods\(^{33}\) using a simulated annealing based method to fit the surrogates both to the data and the hypothesis under consideration. Later in this paper we will refer to the technique developed by Schreiber and Schmitz\(^{28}\) as algorithm 3. Kanjilal and co-workers\(^{34}\) have developed a technique to decompose and shuffle various cycle dependent properties of an almost periodic signal independently. This method tests for more specific hypothesis than those addressed by Theiler’s\(^{31}\) cycle shuffled surrogates. Finally, Small and Judd\(^{30,35}\) proposed a nonlinear model based technique which allows for arbitrary (however not determined \textit{a priori}) hypotheses to be tested. In this paper we concentrate on the three original hypotheses as our immediate goal is to distinguish our data from linearly filtered noise with static nonlinearity.

Theiler and colleagues\(^{26}\) have shown that certain surrogate generation techniques, namely those described in Ref. 25, have a rate of false rejection and false positives consistent with the confidence level ascribed to the test. That is, if one applies the algorithm 2 to generate surrogates at a 95% confidence level, one expects 5% false rejection.\(^{26}\)

Given a statistic \( d \) measured from the raw data, mean \( \mu_d \) and standard deviation \( \sigma_d \) observed from surrogates it is usual to calculate the significance \( s \) of the distinction between data and surrogate. Significance is calculated by
\[
s = \frac{\mu_d - d}{\sigma_d}.
\]
Assuming approximately normally distributed statistic values for the surrogates a value of \( s > 3 \) corresponds to a probability \( p > 0.9987 \) that data and surrogates are different. Note that Eq. (5) is equivalent to applying a one-sided hypothesis test. Since the correlation dimension of a deterministic system should be less than that for noise we expect that \( d < \mu_d \) if the null hypothesis is false.

III. CALCULATIONS

In this section we describe the results we have obtained by applying the methods described in Sec. II to experimental data. Section III A describes the data we utilize in this paper and Sec. III B describes the numerical results we have obtained.

A. Data

Data were collected from a prospective, randomized, double blind, controlled drug intervention study of pig subjects, which was performed at the research laboratories of the University Hospital in Vienna. The experimental protocol was approved by the local institutional animal investigation committee. In pigs of either sex anesthesia was started with ketamin 30 mg/kg intramuscular after venous access thiopental 10 mg/kg intravenous was administered. Then the animals were intubated and piritramid 15 mg and pancuronium 2 mg were given. To maintain anesthesia and relaxation during the experiment piritramid 1.5 mg/kg/h intravenous and pancuronium 0.2 mg/kg/h intravenous were used. The ECGs (lead II) were obtained by means of needle electrodes. The data were recorded at 300 Hz after being digitized with a 12 bit analog to digital converter. After preparation and instrumentation with arterial and venous catheters a pacemaker probe (Vygon GMP 110 cm, 5F, Vygon Corp., East Rutherford, NJ) was inserted via the left femoral vein into the right ventricle and ventricular fibrillation cardiac arrest was induced by applying an alternating current impulse of 90 V and 60 Hz via the pacemaker probe. Cardiac arrest was confirmed by ECG readings and drop of blood pressure. After 5 min of untreated ventricular fibrillation, CPR was started with a mechanical precordial compression device. At 10 min after cardiac arrest, drug treatment was started. At 25 min after cardiac arrest, the animals were externally defibrillated with a series of three repetitive defibrillations to achieve restoration of spontaneous circulation.

From 53 observations 7 were selected, on the basis of visually apparent stationarity, for analysis using the techniques described in this paper. Each selected data set was tested for dynamical stationarity using the space time index plots and near-neighbors tests. All 7 data sets were found to be sufficiently stationary within a time window of 10 000 points. We found that the other 46 data sets not selected on the basis of visually stationarity also failed the algorithmic test for dynamical stationarity. In many data sets the signal exhibited a gradual drift in the first 1–2 s following the onset of VF. The correlation dimension estimation and surrogate techniques we employed require dynamical stationarity. Hence, for each of these 7 recordings we examined the period of early VF from datum 1001 to 9192 (approximately 3.34–30.64 s after the onset of VF). Hence, each recording consisted of \( N_t = 8192 \). Here, an integer power of two was selected for \( N_t \) to speed up the numerical work which involved taking fast Fourier transformations. Figure 1 shows one of the data sets used in this study. Figure 2 shows a representative (algorithm 2) surrogate data set, for comparison.

B. Results

In this section we describe the results of our calculations. We first describe calculations using Judd’s dimension estimation algorithm and then we describe the results of applying the maximum likelihood Grassberger–Procaccia method. From each of the seven data sets 30 surrogates were generated according to algorithm 1 and algorithm 2. (Visual inspection of VF clearly demonstrates that it is distinct from independent and identically distributed noise. Therefore, it was unnecessary to apply algorithm 0.) Each data set (true and surrogate) was embedded with embedding dimension \( d_e = 4,5,6, \ldots,10 \). Each vector time series was then used to calculate correlation dimension according to the method suggested by Judd. Hence, for each of seven data sets, 30 surrogates were generated and embedded to produce 217 embedded time series (including the raw data) and dimension estimates.
Figures 3–5 summarize the results of these calculations for the dimension estimation algorithm described by Judd. Figure 3 shows calculated values of \( s \) for \( d = d_c(\varepsilon_0) \) using algorithm 1 surrogates. Since Judd’s dimension estimation routine provides a function of scale we have that

\[
s(\varepsilon_0) = \frac{\mu_{d_c}(\varepsilon_0) - d_c(\varepsilon_0)}{\sigma_{d_c}(\varepsilon_0)}.
\]

(6)

Each panel in Fig. 3 shows curves of \( s(\varepsilon_0) \) for \( d_c = 4, \ldots, 10 \) as indicated. Figure 4 gives the analogous results from algorithm 2 surrogates. Figure 5 shows estimates of the correlation dimension \( d_c(\varepsilon_0) \) for \( d_c = 4, \ldots, 10 \).

The algorithm 2 surrogate tests shown in Fig. 4 demonstrate a clear distinction (\( s > 3 \) and hence a significance level \( >99.87\% \)) between data and surrogates. In each case this is first evident for a value of \( d_c \) larger than 5. This suggests that lower dimensions are insufficient to successfully embed this data to estimate dimension. In all cases, for some value of \( d_c \) and \( \log(\varepsilon_0) \), the data are clearly distinct from the distribution of surrogate values: We are able to reject the hypothesis that the system is a monotonic nonlinear transformation of linearly filtered noise.

Since we reject the hypothesis associated with algorithm 2 it follows that the hypothesis associated with algorithm 1 must also be rejected. However, the distinction between data and surrogates is not so clear in this case (Fig. 3). However, for all but one data set (h2) the distinction between data and surrogates is sufficient \( [s > 3] \) for some values of \( d_c \) and \( \log(\varepsilon_0) \).

Our dimension estimation calculations for the data indicate that the largest scale structure is around three-dimensional while the smallest ("fractal") structure has a dimension of slightly less than 6. From estimates of correlation dimension according to the technique suggested by Judd we conclude that \( d_c \in (5,6) \). It is interesting to note that the dimension estimation calculations are consistent from subject to subject. In all cases the results appear very similar.

We repeated this analysis using an estimation algorithm based on maximum likelihood estimates of the Grassberger–Procaccia correlation dimension \( D_2 \). In obtaining \( s (d = D_2) \) for a given \( d_c \), we first computed \( s \) at a series of different scales within a scaling window and then simply took their average value. Apart from the algorithms 1 and 2, in this test, we also calculated \( s \) values using the surrogate algorithm 3 proposed by Schreiber and Schmitz,\(^{28}\) the purpose of which was to avoid spurious rejection of null hypothesis. All calculations given in Table I were performed in terms of embedding dimension \( d_c = 12.50 \) surrogates were generated from each algorithm. For algorithm 1, all the \( s \) values are greater than 3 except data set h2. We are able to reject the null hypothesis that the data are a static monotonic nonlinear transformation of linearly correlated noise with a significance level \( p>99.87\% \) for 6 data sets among all 7 sets. The results were confirmed by algorithm 3. We note that the difference is not significant in data set h2.

The results given in Table I are by no means optimal. A previous study\(^{25}\) has shown that the significance \( s \) depends upon the embedding dimension \( d_c \) and a better embedding will lead to a significant discrimination. If \( d_c \) is too low the nonlinear dynamics cannot be unfolded while if \( d_c \) is too high then the large fluctuation in estimated correlation integral (2), and subsequently correlation dimension, due to the sparseness of embedded points will increase the standard deviation \( \sigma_d \) in Eq. (5). In view of this, we have carried out further tests by using different embedding dimensions and data segments within the first 3 min. For the case of low embedding dimensions the significance is increased. For example, the \( s \) values for data set h2 are increased to 2.105 ±0.254, 3.298±0.359, and 3.916±0.416 for the three kinds of surrogates with \( d_c = 6 \).

All of these calculations were also repeated after reducing the sampling rate from 300 to 200 Hz and 100 Hz (with \( N_f = 8192 \) as before). In each case the results were consistent with those presented in this paper.

IV. CONCLUSIONS

Our surrogate calculations provide clear evidence that VF is not a static monotonic nonlinear transformation of linearly filtered noise. Dimension estimates (Judd’s algorithm) suggest that VF has a correlation dimension of slightly less...
Algorithm 1 surrogate calculations from Judd's algorithm. Comparison of data and surrogates for each of the seven data sets using algorithm 1 surrogates and Judd's algorithm. Horizontal axis is the logarithm (to base $e$) of the viewing scale $\log(e_0)$, vertical axis is the number of standard deviations separating data and surrogate as a function of $e_0$. The deviation is calculated as $s(e_0) = \frac{[\mu_d(e_0) - \mu_s(e_0)]}{\sigma_d(e_0)}$ where $\mu_d(e_0)$ and $\sigma_d(e_0)$ are the mean and standard deviation of the surrogate values of $d_s(e_0)$.

FIG. 3. (Color online) Algorithm 1 surrogate calculations from Judd's algorithm. Comparison of data and surrogates for each of the seven data sets using algorithm 1 surrogates and Judd's algorithm. Horizontal axis is the logarithm (to base $e$) of the viewing scale $\log(e_0)$, vertical axis is the number of standard deviations separating data and surrogate as a function of $e_0$. The deviation is calculated as $s(e_0) = \frac{[\mu_d(e_0) - \mu_s(e_0)]}{\sigma_d(e_0)}$ where $\mu_d(e_0)$ and $\sigma_d(e_0)$ are the mean and standard deviation of the surrogate values of $d_s(e_0)$. 

Key

- $d_s=10$
- $d_s=9$
- $d_s=8$
- $d_s=7$
- $d_s=6$
- $d_s=5$
- $d_s=4$
Algorithm 2 surrogate calculations from Judd’s algorithm. Comparison of data and surrogates for each of the seven data sets using algorithm 2 surrogates and Judd’s algorithm. The horizontal axis is the logarithm (to base $e$) of the viewing scale $\log(e_0)$, vertical axis is the number of standard deviations separating data and surrogate as a function of $e_0$. The deviation is calculated as $s(e_0) = \frac{|\mu_d(e_0) - \mu_s(e_0)|}{\sigma_d(e_0)}$ where $\mu_d(e_0)$ and $\sigma_d(e_0)$ are the mean and standard deviation of the surrogate values of $d_d(e_0)$.

FIG. 4. (Color online) Algorithm 2 surrogate calculations from Judd’s algorithm. Comparison of data and surrogates for each of the seven data sets using algorithm 2 surrogates and Judd’s algorithm. The horizontal axis is the logarithm (to base $e$) of the viewing scale $\log(e_0)$, vertical axis is the number of standard deviations separating data and surrogate as a function of $e_0$. The deviation is calculated as $s(e_0) = \frac{|\mu_d(e_0) - \mu_s(e_0)|}{\sigma_d(e_0)}$ where $\mu_d(e_0)$ and $\sigma_d(e_0)$ are the mean and standard deviation of the surrogate values of $d_d(e_0)$.
FIG. 5. (Color online) Correlation dimension estimated by Judd’s algorithm. Correlation dimension estimates for each of the seven data sets using the correlation dimension estimation algorithm described by Judd. The correlation dimension $d_c(\epsilon_0)$ is plotted as a function of $\log(\epsilon_0)$, the logarithm (to base $e$) of the viewing scale.
than 6. This is consistent with our alternative measurements using Taken’s maximum likelihood estimate of the Grassberger–Procaccia algorithm and the Gaussian kernel algorithm.\(^{37}\)

We hypothesize that VF is a complex deterministic process with a small scale noise (high dimensional) component. On the largest scales VF behaves as a nonlinear deterministic system with 3 or 4 degrees of freedom (our scale-dependent dimension estimates provide evidence of this). At moderate to small scales the dimensional complexity increases to around 6. This is consistent with VF having 5 degrees of freedom and additional (possibly chaotic) structure. At the smaller scales the system is likely to be dominated by noise (high dimensional or nonstationary sources). Nonlinear dynamics provides a plausible alternative hypothesis which needs to be examined more closely.

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