Characterizing system dynamics with a weighted and directed network constructed from time series data

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In this work, we propose a novel method to transform a time series into a weighted and directed network. For a given time series, we first generate a set of segments via a sliding window, and then use a doubly symbolic scheme to characterize every windowed segment by combining absolute amplitude information with an ordinal pattern characterization. Based on this construction, a network can be directly constructed from the given time series: segments corresponding to different symbol-pairs are mapped to network nodes and the temporal succession between nodes is represented by directed links. With this conversion, dynamics underlying the time series has been encoded into the network structure. We illustrate the potential of our networks with a well-studied dynamical model as a benchmark example. Results show that network measures for characterizing global properties can detect the dynamical transitions in the underlying system. Moreover, we employ a random walk algorithm to sample loops in our networks, and find that time series with different dynamics exhibits distinct cycle structure. That is, the relative prevalence of loops with different lengths can be used to identify the underlying dynamics. © 2014 AIP Publishing LLC.

Recently, network techniques have been suggested to be useful to characterize features of time series. They provide a range of powerful tools to describe the underlying dynamics in the network domain. A key point of this methodology is the transformation technique from time series to networks by which the information underlying the time series is encoded in the network structure. So far, several transformation approaches have been reported. They construct networks based on time series from different perspectives. In this paper, we introduce a new transformation method, which differs substantially from the existing methods. With our method, proximate states of the underlying system are described by a pair of symbols. We represent each individual symbol-pair as a node and build connections between the nodes to preserve temporal information. Results show that the structures of our networks generated from different dynamical regimes are quite different, and these distinctions can be appropriately quantified by measures commonly used in network science.

I. INTRODUCTION

Graphs, and networks, have been widely used to study the behavior of complex systems with symbolic dynamics.1 However, their direct application to time series analysis was not proposed until fairly recently. Zhang and Small treat cycles in a pseudo-periodic time series as nodes of a network and define links between nodes based on the similarity between cycles.2 With this new representation, some features of time series can be well characterized via a variety of network measurements.3

Since then, several other approaches for transforming time series into networks have been reported. For a review of many of these, see Ref. 4. As described in that review, depending on the way in which one defines nodes and links, the resultant networks can be divided into three broad categories: visibility graphs, proximity networks, and transition networks. Of these three, networks based on the visibility graph algorithm are distinct from the others: these networks are deliberately constructed to capture particular patterns (in particular, self-similarity) in the original time series.5,8 Conversely, proximity networks are constructed based on information related to the reconstructed phase space. In these networks, nodes represent segments of time series or vectors in the reconstructed phase space, and connectivity depends on their mutual proximity. For instance, cycle network,2 correlation networks,9,10 and recurrence networks11–15 are all proximity networks. These networks provide a new perspective for some nonlinear measures related to attractor topology.16 Transition networks are also generated based on connections between distinct states: the amplitude of a measured time series is divided into a finite number of classes (a quantization of phase space) and each class is represented as a node.17–19 Unlike proximity networks, this transformation mainly makes use of the temporal information, i.e., networks are generated according to the transitions between classes. With appropriate parameters, these networks may also be able to roughly capture the fundamental structure of the attractors.4 Moreover, the amplitude-binning method makes the structure of such networks less sensitive to observational noise.

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Very recently, a basic framework for an ordinal partition transformation has been outlined to establish complex networks, which can be used to directly characterize the dynamical evolution of time series.\textsuperscript{20} With this method, segments of a given time series are described by their individual ordinal patterns and each pattern is considered as a node. The temporal order between segments is preserved by drawing links. Simply speaking, the ordinal pattern of a segment is a permutation defined by the order relationship among its elements. And for a given time series, its visible \textit{L}-order ordinal patterns can be obtained by sliding a window of length \(L\) \((L \geq 2)\) and, for every windowed segment, replacing the real values with the order relationship among them.\textsuperscript{21} Essentially, this method provides a simple scheme for phase space partition while avoiding an explicit embedding procedure and ensuring noise robustness—dividing the phase space into some small cells in which the observations of the system ensue the same order structure. By using links to represent their temporal order, ordinal partition networks have the potential to describe the dynamical features of the underlying systems.

It is clear that the idea of ordinal partition transformation has some similarities with the existing transition networks, i.e., states sharing some common attributes are considered as a node, and the temporal order of observations is encoded into network connections. From this point of view, the ordinal partition network is a new kind of transition network. Compared with the amplitude-binning method, one of the advantages of ordinal partitions is that it is better suited to capturing some useful information related to small amplitude variations.

In previous studies of ordinal time series analysis, the parameter \(L\) is usually recommended to be around 3, 4, …, 7.\textsuperscript{22,23} But for continuous systems, it is closely related to sampling rate, which is always quite high due to the consideration of anti-aliasing. Thus, we should vary \(L\) to identify sufficient states. In Ref.\textsuperscript{20}, the values of \(L\) determined by maximal entropy for data from the Rössler system (with various parameter values) are all larger than 14. The resulting networks still cannot adequately distinguish between the different time series. Moreover, large values of \(L\) make results more sensitive to noise. To overcome this problem, one may make use of an alternative form of ordinal patterns, constructed with a time-delay greater than one. But by doing this, one needs to determine a proper method for choosing that time delay.\textsuperscript{24,25}

It is straightforward to see that ordinal patterns, either with time delay or not, are only able to summarize segment shapes—and not magnitude. Amplitude level information between each window has been suppressed.\textsuperscript{26} Nonetheless, this additional information may also provide important detail of the system state—particularly when describing information on a longer time scale. Inspired by this fact, we here use a set of symbols to describe the amplitude levels of the sliding window at different time points, and we combine this level-encoding symbol with the shape-encoding symbol (i.e., an ordinal pattern) to characterize a windowed segment. Actually, the basic idea of this mixed encoding is not entirely new.\textsuperscript{27,28} In Ref. 27, Donner and his co-workers have suggested a mixed form of static and dynamic encoding to define a symbolic recurrence plot, and they applied it to analyze a given event-discrete production model with success. In the current work, on the basis of our mixed strategy, we establish weighted and directed networks from time series to examine the underlying system’s behavior in more detail. To verify the validity of our approach, we apply it to a family of time series from the Rössler system with different parameters. Standard measures in network science are calculated to examine the topological structure of these networks produced from the Rössler system as it undergoes bifurcations. In particular, self-avoiding random walks are used to study the ring structure (or “loops”) in our networks. We find that the topological properties of these networks sensitively trace dynamical changes in the underlying system.

The remainder of this paper is organized as follows: Sec. II introduces the procedures underlying our transformation from time series to networks and some measurements, which we use to characterize the topological features of the weighted and directed networks. In Sec. III, we applied the proposed method to a family of data from the Rössler system. Our conclusion is given in Sec. IV.

II. METHODS

In this section, we begin with a brief review of ordinal partitions of time series. Then, we present our modifications to this standard scheme and introduce our transformation from time series to networks. Finally, some standard measures for characterizing the topological structure of networks are described. In particular, we introduce an algorithm for examining loop structures in directed networks in Sec. II B.

A. Network construction from time series

Given a time series \(\{x_i\}_{i=1}^N\) take a window of length \(L\) \((L \geq 2)\) sliding along this series and denote the windowed segment at time \(t\) as \(X_t = \{x_t, x_{t+1}, …, x_{t+L-1}\}\). The ordinal pattern \(\pi_L(t) = (\tau_1, \tau_2, …, \tau_L)\) of \(X_t\) is defined as the permutation of \((1, 2, …, L)\) satisfying \(x_{(t-1)+\tau_1} \leq x_{(t-1)+\tau_2} \leq … \leq x_{(t-1)+\tau_L}\). In order to obtain unique results, we take \(x_i \leq x_j\), if \(i < j\), in case of \(x_i = x_j\). For instance, the series shown in Fig. 1(a) has \(x_1 = -3.385, x_2 = -3.21\) and \(x_3 = -2.884\). To get \(\pi_1(1)\), one rearranges the values in ascending order \((x_1, x_2, x_3)\), and thence obtain \(\pi_1(1) = (1, 2, 3)\). This definition has inherent similarities with time-delay phase space reconstruction (embedding dimension equals \(L\), time delay is 1). In time delay embedding theory, points in the reconstructed phase space are treated as distinct states of the underlying dynamical system. However, by using ordinal patterns, states are considered to be the same when two vectors have the same order structure—essentially creating an equivalence class of states.

Intuitively, this ordinal partition only makes use of the shape information of segments. For typical high-dimensional continuous systems, this representation may be too coarse and ignore many significant details. For example, Fig. 1(b) shows a trajectory of the (nonetheless fairly low-dimensional) Rössler system determined by equation
The bifurcation parameters $a = 0.406$, $b = 2$, $c = 4$. We observe the values of the $x$ component for several time units and highlight the corresponding states of this system in state space, as shown in Fig. 1. It is easy to see that the patterns of some windows, when the states of the system are indeed quite different, could still share a common order structure, such as, when $t = 16$ (marked by a black dot) and $t = 48$ (a green dot), even $L = 20$.

To better capture more details of the system behavior (while maintaining the useful features of the ordinal representation), we propose a simple modification of adding amplitude information. Specifically, a pair of symbols are used to describe the segment in a window. One symbol describes the amplitude level and the other is the ordinal pattern, as introduced above, which summarizes the segment’s shape. The former symbol is obtained by splitting the range of the time series $[a, b]$ into $Q$ equal regions. Each region is labeled by an index. The amplitude level of a segment in window $X_i = (x_i, x_{i+1}, \ldots, x_{i+L-1})$ is then described (quantized) by the index of the bin within which each point $x_i$ falls.

As illustrated in Fig. 1(a), we first find the maximum $\beta_{\text{max}}$ and the minimum $\beta_{\text{min}}$ of this series, and then divide equally the range from $\beta_{\text{min}}$ to $\beta_{\text{max}}$ into $Q = 6$ regions, each of which is labeled with a symbol. For example, the level-encoding symbol for window $w = 16$ is denoted as $x_l$. By combining with its own ordinal pattern $\pi_5(16) = (5, 4, 3, 2, 1)$ (when the window length is 5), the windowed segment is symbolized as $S(16) = (x_l, \pi_5(16))$. Similarly, the segment in window $t = 48$ is recorded as $S(48) = (x_l, \pi_5(48))$, $\pi_5(16) = \pi_5(48)$. This modification provides a basis for our new phase space partition.

To investigate the transitions among the different states identified by our modification, a weighted and directed network is constructed with fixed $Q$ and $L$ as follows: every symbol-pair visible in a given time series is represented by a node; their temporal succession are preserved by drawing links. Naturally, there are two feasible ways to build the corresponding connections: non-overlapping, that is the link starting from the node corresponding to $S(t)$ ends at the node representing $S(t + L)$, $t = 1, 2, \ldots, N - 2L + 1$; while for the case of overlapping, it ends at the node corresponding to $S(t + 1)$, $t = 1, 2, \ldots, N - L$. In this study, we just make use of the later. The weight $w_{ij}$ of the link directed from node $i$ to $j$ is given by

$$w_{ij} = \#(S^i \rightarrow S^j),$$

where $S^i, S^j$ are the symbol-pairs corresponding to node $i, j$, respectively, and $\#(S^i \rightarrow S^j)$ is the number of times that the transition from $S^j$ to $S^i$ occurs in a given time series.

Finally, the adjacency matrix $W = (w_{ij})_{M \times M}$, $w_{ij} \geq 0$ is used to represent the generated networks. $M$ is the network size. If $w_{ij} > 0$, then there is a link with weight $w_{ij}$ from node $i$ to $j$.

### B. Structure measures in complex networks

Before applying our networks to study dynamical systems, we first describe several standard measures for the characterization of network structure. Here, we continue to use $W = (w_{ij})_{M \times M}$, as described above, to represent the obtained network.

First, consider two basic statistics: degree and strength. The degree $k_i$ of a node $i$ is defined as the number of links that connect to it. In a directed networks, $k_i = k_{\text{out}}^i + k_{\text{in}}^i$, where $k_{\text{out}}^i$ is the number of outward links from node $i$ and $k_{\text{in}}^i$ is the number of inward ones. Usually, researchers use the degree distribution to gain insight into the network structure. Besides that, some corresponding scalar measures are suggested to be useful to quantify the network globally, such as the first and second moment of the degree distribution. The variance of degree sequence is considered as an indicator of the network heterogeneity, and it can be calculated as follows:

$$\sigma = M^{-1} \sum_k k^2 M(k) - \langle k \rangle^2,$$

where $\langle k \rangle$ is the average degree, and $M(k)$ is the number of nodes, which have degree $k$. The strength of a node is the sum of the weights of its links. Analogously, we use $s_{\text{out}}^i$ and $s_{\text{in}}^i$ to denote the out-strength and in-strength of node $i$. According to our transformation, usually $s_{\text{out}}^i \approx s_{\text{in}}^i$. It is...
straightforward times of the corresponding symbol-pair occurring in the given time series. So measures based on the distribution of visible symbol-pairs can be easily derived from the adjacency matrix $W$.

Moreover, we can also characterize networks by their link densities. The link density $\rho$ is defined as the ratio of the number of links $N_{\text{link}}$ to the maximum possible number of links, given by

$$\rho = \frac{N_{\text{link}}}{M(M-1)}.$$  \hspace{1cm} (4)

In a weighted and directed network, the shortest path from node $i$ to $j$ is defined as the path from node $i$ to $j$ with the lowest cost. The cost (here, the sum of the link weights) is defined to be the shortest path length ($SPL$) from node $i$ to $j$. We should note that, in our networks, the link weight $w_{ij}$ is not a distance but actually a frequency with which the symbol-pairs corresponding to node $i$ and $j$ appear in succession. Hence, the transition matrix $R = (p_{ij})_{M \times M}$ of these patterns can be easily derived from $W$ and

$$p_{ij} = \frac{w_{ij}}{\sum_{l=1}^{M} w_{il}}.$$  \hspace{1cm} (5)

Then, we define the distance $d_{ij}$ from node $i$ to node $j$ as the $1/p_{ij}$. With this definition, if the distance from one node to another is quite small, that means their corresponding states often occur in succession. So, the $SPL$ between two nodes here might relate to the time interval taken by trajectory to visit their corresponding region in phase space.

In addition to these measures, the network structure also can be characterized by the statistical distribution of loop (i.e., closed paths) lengths $\zeta$. According to our transformation, it is easy to see that the loop structures in our networks have a specific dynamical meaning. Loops correspond to the system returning to a state similar to a former state (a state recurrence). Hence, in this work, we use the loop length distribution to study the recurrence time distributions from the network structure.

Theoretically, all the loops in a network can be found with the help of depth-first algorithms. However, this is feasible only for fairly small networks with relatively sparse connections. By employing a Monte Carlo procedure,\textsuperscript{31} sampling loops by random walks may offer us a practical method to study the distribution of loop lengths in larger and denser networks. Although the procedure in Ref. 31 is performed on undirected network, it is easy to generalize to directed networks.

The process is described as follows. We iteratively remove nodes whose out-degree is zero until the network size no longer changes. After that, we perform repeated (and numerous) random walks, starting from a selected node $v_0$, to sample loops in the given network. At every time, (1) we first search for the set $V(0)$ consisting of all the nodes, each of which has a link directed from the starting node $v_0$; (2) we randomly select a node $v_1$ from $V(0)$. If $v_1 = v_0$, a loop with length $\zeta = 1$ is obtained (self-loop), else we mark $v_1$ as the second node of this random walk; (3) further, we search for the neighbor set $V(1)$ of node $v_1$ and repeat the steps above; (4) finally, when the walk returns to the starting node, a loop is recorded together with its length $\zeta$. In this process, attention should be paid to avoiding any self-intersection (and, hence, the discovery of a shorter loop). That is, neighbor set obtained from the current node does not contain the nodes that are already determined by this walk (apart from the starting one). By repeating the procedure for many times, an estimate of the loop length distribution can be obtained.\textsuperscript{32}

III. APPLICATION TO DATA FROM DYNAMICAL MODELS

In this section, the Rössler system (see Eq. (1)) is used as a benchmark example to show the validity of our method. We select $b = 2$, $c = 4$ and $a \in [0.37, 0.43]$ with a step size $\Delta a = 0.0001$. With these values of $a$, the system exhibits various dynamical regimes and undergoes the period-doubling route to chaos as $a$ is increased.\textsuperscript{33} Here, we applied our network construction method to time series from this model. We expect that it can capture useful information from the underlying dynamical system. Thus, some details, such as the distinctions between different dynamical regimes and the changes in system dynamics, can be quantitatively described by means of network methods.

For every $a$ here, we calculate a numerical solution for this system via the fourth/fifth order Runge-Kutta technique (the integration step is 0.2) with an initial condition $(x_0, y_0, z_0)$ randomly selected in $[0, 1] \times [0, 1] \times [0, 1]$, and then we record $2 \times 10^4$ successive values of $x$ component after removing the leading $10^4$ observations (to eliminate transient states). In our simulation, these 601 series are used to study the performance of our approach. Besides that, we take a time series generated with $a = 0.3$ (period-1) to demonstrate the results obtained from low-order periodic dynamics. Note that in continuous system analysis, people refer to period-$n$ with $n$ indicating the number of times that an orbit winds around before closing.

First, we use the proposed method to transform every time series into a network. It is found that networks generated from different dynamical regimes exhibit different characteristics. For the present paper, we display four representative networks constructed from four different dynamical regimes: period-1, period-4, period-8, and chaos (Fig. 2). The chaotic dynamics we consider here are simply the archetypal chaotic Rössler system. Differences in results between phase coherent and funnel regime dynamics, for example,\textsuperscript{34} may also provide significant results. As shown in Fig. 2, the topological structure of each differs significantly. In the case of period-1, the resultant network possesses a ring-like structure. With the increasing complexity of the underlying system, the networks’ structure become more and more complicated, see Figs. 2(b)–2(d). And we find that this fact is corroborated when the networks are generated with larger $Q$ and $L$. In this work, we fix $Q = 100$, $L = 8$. Of course, varying $Q$ and $L$ will affect the total number of nodes in the network. However, as the relationship is trivial and not
all potential nodes actually occur in the network, we cannot provide a precise formula.

For further analysis, in Fig. 3, we display the degree distribution for those four networks. Although there seems to be no universal function for their distribution, it is clear that networks obtained from chaotic data have some nodes with higher degree and a high level of heterogeneity $\sigma = 29.9$. On the contrary, for networks representing periodic data, the degree is always small, and so is the degree of heterogeneity. In particular, in the case of period-1 (Fig. 3(a)), the degree of most nodes are under 6 and the level of heterogeneity $\sigma = 0.88$. Analogous analysis can be performed on the out-strength distribution, as shown in Fig. 4. The histograms here are constructed with a fixed bin width 10. The distributions of these two basic statistics provide a clear demonstration of the distinctions among the networks derived from different dynamical regimes. So, their corresponding scalar measures may provide several effective tracers for the dynamical changes of the underlying system.

In previous studies, significant effort has been expended to identify the dynamical transitions of a continuous system. In the following discussion, we demonstrate how to address this problem by investigating measures of networks constructed by our new transformation.

The bifurcation diagram in Fig. 5(a) gives us an intuitive feeling for the behavior of the Rössler system on the interval
$a \in [0.37, 0.43]$. It should be mentioned here that the diagram is drawn by using the *extrema* function (in the TISEAN package), which corresponds to placing a Poincaré section determined by zeros of the derivative with respect to $x$. As shown in Fig. 5(a), this system starts with a low-order periodic motion, and then becomes chaotic through a sequence of period doubling bifurcations as $a$ increases. While $a$ reaches 0.4088, another low-order periodic (period-3) window appears. After that, it quickly enters more complicated chaotic regimes. Furthermore, scrutinizing the chaotic regime, several periodic windows can be found. Here, their positions are marked with red lines in Fig. 5, including: $a=0.3908$ (period-12), $a=0.3999$ (period-20), $a=0.4032$ (period-7), etc.

FIG. 4. The out-strength distributions of the complex networks shown in Fig. 2.

FIG. 5. (a) The bifurcation diagram of the Rössler system, when $a$ in Eq. (1) is varied from 0.37 to 0.43. The vertical axis is the local maximum values of the $x$ component; (b)–(f) values of the average out-strength, the variance of the degree sequence, the average degree, the network density, and the average SPL of networks obtained with different $a$. 
Now we calculate the average out-strength, the variance of the degree sequence and some other basic network measures for analyzing bifurcations and chaos on the interval \( a \in [0.37, 0.43] \). As shown in Fig. 5(b), the value of average out-strength is sensitive to the presence of dynamical transitions in the underlying system. In this figure, the occurrence of periodic windows is clearly identified by high values. Moreover, different kinds of chaotic regimes can be distinguished as well: chaos with higher complexity results in smaller average out-strength. Due to the influence of network size, some chaotic time series may have slightly smaller average degree than the higher-order periodic ones (Fig. 5(d)). Actually, the detailed structures of the corresponding networks are quite distinct, and this is reflected by the variances of their degree sequences, as given in Fig. 5(c). This demonstrates that networks generated from time series produced during chaotic regimes are more heterogeneous. There are also some straightforward applications of other scalar network measures, such as the average SPL and the density, as shown in Figs. 5(e) and 5(f). From the results above, we conclude that our approach can be used to discriminate different dynamical regimes and to detect dynamical changes in complex systems.

Furthermore, some other network characteristics also provide interesting results that show the potential of this method for time series analysis. We use the method introduced in Sec. II B to investigate the loop length distribution in our networks. The link direction defined in our networks makes the procedure of sampling loops much simpler than typically considered.\(^{37}\) Moreover, constrained by directedness, several large loops will typically emerge. In order to sample loops as broadly as possible, we select the node with the largest degree as the starting node and then repeat these experiments many times (In the present work, \( 5 \times 10^3 \) times). This is much larger than our networks’ size, which is always smaller than 2000. From this, we find that further repetition will not change the features of the loop length distribution.

Figs. 6(a)–6(d) display the results obtained from networks shown in Figs. 2(a)–2(d), respectively. It is easy to see that the distribution of loop lengths in these networks has characteristic features. Both multi-periodic and chaotic dynamics can result in a multi-modal distribution. However, in the networks derived from periodic time series, clear gaps are observed between individual modes—that is, loops of some certain lengths do not exist. Moreover, although the positions of peaks do not give the exact number of points in a period (periodic data) or a pseudo-period (chaos), it seems to be closely related to the periodicity (the period, or pseudo-period, is about 0.62 s or about 31 points).

Now we illustrate the relation between the loop length distribution and trajectory structure with two examples. As shown in Fig. 7(a), we first select a loop (with length 26) from its corresponding network and find the corresponding cell of every node in phase space. It is straightforward to see that the cell determined by one node may be visited several times during our observation. Here, we randomly select one of these visitations to mark out the cell’s position (red dots) on the trajectory. Arrows are used to indicate the direction of the selected loop. From Fig. 7(a), we find that the loop behavior is always constrained to the orbits. That is, the loop behavior is essentially phase-locked to the underlying periodic attractor. This is why we observe both significant peaks and gaps in the loop length distribution. However, in the case of chaos (Fig. 7(b)), due to the denseness of chaotic trajectories, a random walk performed on the corresponding network can come back to the starting node via numerous different paths. This may result in a much broader range of the loop lengths; and in this range, loops of any length are possible.

When adding dynamical noise to the periodic system, the periodicity is quickly obscured. Fig. 8(a) shows the distribution of loop lengths in the networks constructed from data of period-1 with additive Gaussian dynamical noise at standard deviation 0.05. As the noise is dynamic, it is

![FIG. 6. The loop length distributions of the networks constructed from time series with different dynamics: (a) period-1, \( a = 0.3 \); (b) period-4, \( a = 0.375 \); (c) period-8, \( a = 0.3837 \); (d) chaos, \( a = 0.406 \).](image-url)
injected into the systems dynamics at the specified sampling rate and then integrated along with the system. That is, the dynamical noise is additive, but it modifies the actual system state and thereby affects the observed trajectory. Compared with the distribution in Fig. 6(a), it can be seen that some longer loops appear, similar to the situation for chaos. But with the same repeating time, only a few loops have been collected. This effect becomes more obvious when the noise level increases, as shown in (b). Nonetheless, the mode-gap is still evident. From this point of view, our method retains potential to distinguish various dynamics described above in the presence of significant noise.

IV. CONCLUSIONS

We have proposed a new method for transforming time series into networks. We modified the popular scheme for extracting ordinal patterns from time series with the addition of amplitude level information. With this technique, fixed length segments in a given time series are characterized by a pair of symbols. We map every individual symbol-pair into a node and then link different nodes according to the temporal information in the time series. By using a typical dynamical model, we have illustrated the potentials of our networks for time series analysis. Simulation results show that some statistics commonly used in network science are sensitive to dynamical transition. In particular, an algorithm based on self-avoiding random walks was introduced to investigate the loop length distribution in our networks. We find that the latter has a close relationship to the underlying dynamical regime.

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21. That is, replace the $L$ numbers $x_1, x_2, x_3, ..., x_L$ with a permutation of the integers $1, 2, 3, ..., L$ such that the $i$th integer indicates the relative size of $x_i$ among the $L$ numbers.


32. We note that we do not claim that this estimate of the underlying distribution is unbiased. We will show that it is enough that it depends in some non-trivial way on the true distribution—and hence the true recurrence pattern of the underlying dynamical system.


