Tests of the random walk hypothesis for financial data

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Abstract

We propose a method from the viewpoint of deterministic dynamical systems to investigate whether observed data follow a random walk (RW) and apply the method to several financial data. Our method is based on the previously proposed small-shuffle surrogate method. Hence, our method does not depend on the specific data distribution, although previously proposed methods depend on properties of the data distribution. The data we use are stock market (Standard & Poor’s 500 in US market and Nikkei225 in Japanese market), exchange rate (British Pound/US dollar and Japanese Yen/US dollar), and commodity market (gold price and crude oil price). We found that these financial data are RW whose first differences are independently distributed random variables or time-varying random variables.

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1. Introduction

The aim of this paper is to examine whether financial data are random walk (RW). To accomplish this, we propose a method based on the small-shuffle surrogate (SSS) method [1]. The novelty of our work is that our method does not depend on the specific data distribution, although previously proposed methods depend on properties of the data distribution, and our investigation is from the viewpoint of deterministic dynamical systems. This paper may be divided into two main parts. The first part considers the methodology and the second part is the application of the proposed method to numerical examples and actual financial data.

A venerable and challenging question is whether financial data are a RW [2]. In the year 1900 Bachelier introduced a RW model of market price [3, Chapter 2] (for more details on the relationship between RW, financial data, and efficient market hypothesis, see elsewhere [4–7] for example). After his pioneering work, numerous studies have been done (for example, see Refs. [6,7]). However, one of two possible results are obtained. That is, data are RW [5] or the data are not [4]. As a result, the question of whether financial data are RW is still controversial. One of the large problems is that previously proposed methods are developed under the assumption that the first difference data (the disturbance term, see below) are generated from independent and identically distributed (IID) noise or Gaussian random numbers (GRN) [8,9]. Methods that test for Gaussian disturbances, or even IID disturbances are useful, but also very restrictive [10]. As is well...
known, data are not always normally distributed [4,11] and often do not even seem to be IID because the volatility of the data is not constant and clearly increases and decreases over time. (see Figs. 7, 10, and 12 in Section 6). That is, these methods to test for RWs are theoretically incompatible and not suitable for practical uses.

In this paper we propose a much more general test against a physically more common null hypothesis (NH) from the viewpoint of deterministic dynamical system. As mentioned above, the volatility of first difference data is not constant and clearly increases and decreases over time. This implies that there might be dynamics of the variance and the volatility may be influenced because of some external factors. However, the original data might be IID random variables. Then, the data become independently distributed (ID) random variables or time-varying random variables. Hence, we consider that it is most important to investigate whether temporal correlations are absent in the data. Our method tests observed time series data against the hypothesis that it may be modelled as a RW with independently, but not necessarily identically, distributed random disturbances. In most experimentally observed systems that may plausibly exhibit RW dynamics, this is the hypothesis of greatest interest. Experimentally one usually cannot assume that data are drawn from a stationary distribution. Moreover, our hypothesis certainly encompasses the more restrictive tests that have been described previously [8,12].

We apply our method to several financial data in different markets: daily stock market (Standard & Poor’s 500 (S&P500) in US market and Nikkei225 in Japanese market), daily exchange rate (British Pound/US dollar (GBP/USD) and Japanese Yen/US dollar (JPY/USD)), and commodity market (daily gold price and daily crude oil price). The analysis of these data is the major novel contribution of this paper.

In Section 2 we briefly review the basic model of RW, current technologies that have already been proposed and the relative problems. In Section 3 the small-shuffle surrogate algorithm and hypothesis will be discussed. Our method is based on this algorithm. In Section 4 we present our choice of discriminating statistics and describe how to test our NH. In Section 5 we apply our method to numerical examples and in Section 6 we apply the method to financial data.

2. Current technologies

Basically the model of a RW can be given by

\[ x(t + 1) = x(t) + \eta(t), \] (1)

where \( \eta(t) \) is called the disturbance term and usually considered to be IID noise or GRN [2,13]. As shown in Eq. (1), the RW \( x(t) \) is simply the sum of the disturbance term \( \eta(t), \tau < t \). That is, a RW is a stochastic process which changes by steps and any step does not depend on the previous history of the data. Hence, the data are unpredictable. As Eq. (1) shows, the structure is obviously very simple, however, the model shows very rich behavior. The irregular fluctuations of RW data can exhibit the appearance of rising and falling behaviors of trends. Hence, when the data shows irregular fluctuations and some trends, we may suspect that the data might be RW.

Usually, to investigate whether the data are RW, the first difference data (that is, \( x(t + 1) - x(t) \)) are used. The difference data should be identical to the disturbance term and also should be GRN, if the system is essentially the same as Eq. (1). In other words, only when data are RW, the difference data can be GRN. Hence, if the difference data are GRN, it is good evidence that the data are RW. However, even if the data are normally distributed, this fact does not necessarily imply that the data are random variables. Several methods have been proposed to investigate whether data are GRN. One of the most commonly employed statistics is the variance ratio (VR) tests [8]. If a time series follows a RW, in a finite sample the increments in the variance are linear in the observation interval. That is, the variance of difference data should be proportional to the sample interval. Another popular statistic is the Hurst exponent [12]. If a time series is a RW, past increments in displacement are correlated with future increments. These correlations are quantified by the Hurst exponent and a RW corresponds to a value of 0.5 [14]. These methods have been applied to a large number of real world data, for example Refs. [6,14]. It should be noted that both these statistics are obtained using features of the

1The volatility changes with time. To express this phenomenon, we use the term “time-varying”.
GRN. That is, these methods depend on data distribution. There is another approach. If we can show that difference data are IID random variables, this also can be an evidence that the data are RW. To investigate directly whether data can be fully described by IID random variables, the random-shuffle surrogate (RSS) method has been proposed [9]. Moreover, we note that building models may also prove useful [15,16]. When the model selected as the best model has the form of Eq. (1), this fact can indicate that the data are consistent with a RW. However, this approach is theoretically effective only when the first difference of the data are IID or GRN.

However, this is not always so. It is well known that the volatility of the first difference data is not constant and clearly increases and decreases over time. That is, the data are not “IID” random variables. Moreover, it is also known that the data are usually not normally distributed [4,11]. Hence, there are several approaches to use the data effectively. The data are often replaced by their logarithm because such data are then approximately normally distributed (that is, the original data are log-normal distributed) [17]. Hence, the data are performing a geometric Brownian motion. However, it is found that the tails of measured distributions are fatter than expected for a geometric Brownian motion [4,17]. Hence, several alternative methods to geometric Brownian motion have been proposed such as the Lévy stable non-Gaussian model, and the Student’s $t$-distribution [17]. It is difficult to treat the data distribution, and preferable to apply a method which does not depend on it.

Since the volatility of the first difference data increases and decreases over time, we consider that although data are originally IID random variables, there are dynamics of the variance or the volatility may be influenced by some external factors. Then, the data become time-varying random variables. If our expectation is correct, the data index (order) itself has no meaning and temporal correlations in the data are absent as well as IID random variables. Hence, we consider that the essential feature of a RW is that the disturbance term is a time-varying random variable. Furthermore, we note that even if the original data are Gaussian, data whose volatility is varying are no longer Gaussian. In this case, the structure itself of Eq. (1) is preserved, but the volatility of the disturbance term changes. This idea urges us to generalize the definition of RW. For time series we should consider that the data are RW if the disturbance term is time-varying random variables. Hence, we consider that the most important thing to investigate whether data are RW is to investigate whether the first difference data are time-varying random variables. We can then conclude that the data are RW.

To tackle this problem we propose a method based on the SSS method. Surrogate methods provide a rigorous test of whether or not the data belongs to a particular class of systems. One can then be assured (or at least persuaded) that the data are amenable to that particular class of models, or not. In this paper our primary interest is in deterministic dynamical systems and we want to identify (and eliminate) wide classes of stochastic process that are not appropriate for study via our deterministic models. Surrogate methods have been proposed to investigate features of the data from the viewpoint of deterministic dynamical systems [9].

3. A different algorithm: the SSS method

To investigate whether temporal correlations in data are absent or data are IID random variables, the SSS method is useful [1]. The SSS method can be applied to data, irrespective of whether the data have trends. Furthermore, the method does not depend on the data distribution. The SSS method has proven to be effective for tackling data exhibiting short term variabilities and long-term trends [1,18,19].

SSS data are generated as follows. Let the original data be $x(t)$, let $i(t)$ be the index of $x(t)$ (that is, $i(t) = t$, and so $x(i(t)) = x(t)$), let $g(t)$ be GRN, and $s(t)$ will be the surrogate data:

(i) Obtain $i'(t) = i(t) + A g(t)$, where $A$ is an amplitude (adding GRN to the index of the original data).

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2One should remember the relation between stationarity and data distribution. In Ref. [17], there is a nice discussion on stationarity of price changes and probability density function. However, we do not wish to overinterpret that relationship.
Monte Carlo hypothesis testing and inspect whether the estimated statistics of the original data fall within or not.

4.2. Monte Carlo hypothesis testing

We have found that choosing $A = 1.0$ is adequate for nearly all purposes, and data are swapped around the neighbor only [1]. In the SSS data local structures or correlations in irregular fluctuations (short term variability) are destroyed and the global behaviors (trends) are preserved. Hence, the SSS data are very similar to the original data. Further details of the mechanism are provided in Refs. [18,19]. The NH addressed by this algorithm is that irregular fluctuations (short term variability) are ID random variables (in other words, there is no short term dynamics or determinism) [1,18,19]. Hence, the SSS method is useful to investigate whether or not the data are time-varying random variables.

To investigate whether data is consistent with a RW we apply the SSS method twice, to the original data and to the first difference data. As Eq. (1) shows, the RW have dynamics, and RW also show irregular fluctuations. Hence, the irregular fluctuations of the original data should have dynamics. In the first application we apply the SSS method to investigate whether irregular fluctuations have dynamics. If we detected that irregular fluctuations have dynamics, we expect that there is a possibility that the original data are RW. When data are RW, the first difference data should be time-varying random variables (ID random variables). In the second application we apply the SSS method again to the first difference data to investigate whether the first difference data are time-varying random variables. If we detect that this is the case, we can consider that the original data are RW. It should be noted that NH of the SSS method is not that of RW. As described above, we develop our method based on the SSS method and apply it to the original data and also to the first difference data. That is, we take advantage of features of RW and the SSS method to investigate whether data are RW.

4. When to reject a NH

Discriminating statistics are necessary for hypothesis testing. After the calculation of the statistic, we need to inspect whether the NH should be rejected or not.

4.1. Discriminating statistics

Discriminating statistics are necessary for surrogate data hypothesis testing. The SSS method changes the flow of information in the data. Hence, we choose to use the auto-correlation (AC) function and the average mutual information (AMI) as discriminating statistics. The AC is an estimate of the linear correlation in data, the AMI is a general nonlinear version of the AC. They can answer the following question: on average, how much does one learn about the future from the past [20]?

It is widely observed that estimating AMI is difficult [21]. The major reason is that it is not easy to estimate the underlying probability distribution reliably. To reduce this problem a new method has been proposed where an adaptive partition is applied [22]. However, the SSS data have the same probability distribution (rank distribution) as the original data. In this case, we consider that the influence due to using different data (the original data and the SSS data) for estimating the joint probability distribution is not large, and we find that there is no significant bias between the estimated joint probability distribution of the original data and the SSS data. Hence, we expect that it is straightforward to compare the AMI of the original data and the SSS data.

4.2. Monte Carlo hypothesis testing

After the calculation of these statistics, we need to inspect whether a NH shall be rejected. We employ Monte Carlo hypothesis testing and inspect whether the estimated statistics of the original data fall within or outside the statistical distribution of the surrogate data [23]. When the statistics fall within the distribution of

\[i(t) \text{ by the rank-order and let the index of } i(t) \text{ be } \hat{i}(t) \text{ (rank-order the perturbed index).}
\]
\[(iii) \text{ Obtain the surrogate data } s(t) = x(\hat{i}(t)) \text{ (reorder the original data with the perturbed index).}^{3}\]
the surrogate data, we conclude that the hypothesis may not be rejected. In this paper, we generate 99 SSS
data and hence the significance level is between 0.01 and 0.02 for a one-sided test with two nonindependent
statistics.4

It should be noted that although the multiple comparison problem is common in surrogate data
applications, we use the AC and the AMI as complementary statistics. This is because some of the test systems
are robust to one or the other of our two primary statistical test (the AC and the AMI). For example, using the
Logistic map data, where the Logistic map is given
\[
x(t + 1) = 4.0x(t)(1 - x(t))
\]
we use 5000 data points and the data is noise free [1]. We note that the Logistic map has clear nonlinear dynamics. However, as Figs. 1(a)
and (b) show, the AC of the original data falls within the distribution of the SSS data, although the AMI of the
original data falls outside the distribution of the SSS data. It is well known that the Logistic map is a nonlinear
system and that its variability is equivalent to that of IID random variables. Hence, these results are consistent
with the known features of the Logistic map. This result indicates that only one statistic (the AC or the AMI)
is not enough for some cases and also implies that there is some possibilities that the AMI does not work for
some cases. Hence, to avoid this problem we adopt two discriminating statistics, the AC and the AMI.
Furthermore, we note that when data are time-varying random or ID random, both the AC and the AMI of
the data must fall within the distribution of the SSS data [1].

Also, we show plots of both the AC and the AMI as a function of time lag. This also appears to be a
multiple comparison. However, in all cases the hypothesis testing is robustly conducted for fixed small values
of lag. In fact, we expect that it is only a meaningful test statistic for small lag, because the AC and the AMI of
the original and surrogate data will coincide for large lag. The plots of the AC and the AMI as a function of
lag are provided for information only.

5. Numerical examples

We now demonstrate the application of our idea, and confirm our arguments with several examples.
To achieve similar situations of actual financial data we use two models. One is that we use a model given by
Eq. (1). The other is that we use a stochastic volatility (SV) model, the Heston SV model [24]. In each case
the number of data points is 10 000.

5.1. The case of using a RW model of Eq. (1)

We first use data generated by Eq. (1). To achieve similar situations that the volatility of the first difference
data increases and decreases over time, we change the standard deviation (SD) of GRN between 0.2 and 1.0 at
random every 400 times, which is used as the disturbance term. Using the disturbance term, RW data are
generated. In the investigations, although RW data show irregular fluctuations, we add GRN as another
(independent) irregular fluctuation or observational noise to the data, where the additional GRN are different
from GRN used for the disturbance term. The reason we investigate only the case of GRN is that although
GRN are white noise, the difference data are not (that is, this is the same case as when data have dynamics).
Also, we do not investigate when the disturbance term has dynamics or is IID noise, because it has been
already confirmed that the SSS method can detect these [1]. We use several different levels for additional
Gaussian irregular fluctuations with SD = 0.0 (that is, noise free), 0.05, 0.1, 0.15, 0.2, 0.5 and 5.0. Fig. 2 shows
the behaviors. Both Figs. 2(a) and (c) show irregular fluctuations and some trends. Fig. 2(b) shows that the SD
of the difference data is not constant, increases and decreases. Fig. 2(d) shows that the SD of the difference
data seem to be constant. This is because the influence of additional GRN is more significant than that of RW.

As an initial investigation and a form of “sanity check”, we first apply the SSS method to the original data
(not the difference data). As Eq. (1) shows, the RW have dynamics, and RW also show irregular fluctuations.
Hence, the irregular fluctuations should have dynamics. However, if the SSS method fails to indicate this, then
it is not appropriate to approximate the data as RW. This is the case when the influence of the additional

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4The significance level of each test is 0.01. If two statistics are identical (dependent), the significance level for the proposed test is 0.01.
If the statistics are independent, the significance level for the test is given by $1.0 - 0.99 \times 0.99 = 0.0199$. The reality should be somewhere
in-between.
Gaussian fluctuations is larger than that of RW data. In this case, our idea clearly fails to identify a RW. On the other hand, if the influence is smaller, the SSS method will indicate that the irregular fluctuations have dynamics.

The results are shown in Table 1. We first see the result of the initial investigation, where the original data are used. When SD is smaller than 5.0, as shown in Figs. 3(a) and (b), both the AC and the AMI of the original data fall outside the distribution of the SSS data. When applied to the original (not difference) data, we can reject the NH. From this result, we expect that the irregular fluctuations have some kind of dynamics and there are some possibilities that the data are RW. However, when SD = 5.0, as shown in Figs. 3(c) and (d),
both the AC and the AMI of the original data fall within the distribution of the SSS data. Hence, we cannot reject the NH. We will conclude at this stage that the data are indistinguishable from RW.

Based on this result, we apply the SSS method again to the first difference data when SD is smaller than 5.0. When SD are smaller than 0.2, as shown in Figs. 4(a) and (b), both the AC and the AMI of the difference data fall within the distribution of the SSS data. Hence, we cannot reject the NH. This result indicates that the difference data are time-varying random variables (ID random variables), then we expect that the original data are RW. When SD is not less than 0.2, as shown in Figs. 4(c) and (d), both the AC and the AMI of the difference data fall outside the distribution of the SSS data. Hence, we will conclude that it is not appropriate to approximate the original data as RW.

### Table 1
The result of testing for RW

<table>
<thead>
<tr>
<th>SD of irregular fluctuations</th>
<th>Initial investigation using original data</th>
<th>Test for RW using difference data</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>R</td>
<td>NR</td>
</tr>
<tr>
<td>0.05</td>
<td>R</td>
<td>NR</td>
</tr>
<tr>
<td>0.1</td>
<td>R</td>
<td>NR</td>
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<tr>
<td>0.15</td>
<td>R</td>
<td>NR</td>
</tr>
<tr>
<td>0.2</td>
<td>R</td>
<td>R</td>
</tr>
<tr>
<td>0.5</td>
<td>R</td>
<td>R</td>
</tr>
<tr>
<td>5.0</td>
<td>NR</td>
<td></td>
</tr>
</tbody>
</table>

The R indicates that the NH of the SSS method is rejected and the NR indicate not rejected. When SD of additional noise is smaller than 0.2 which is minimum SD value of GRN used as the disturbance term, we can identify data as RW. However, when this is not the case, we fail to do so.

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Fig. 3. (Color online) A plot of the AC and the AMI for simulated RW data: (a) and (b) SD = 0.5, and (c) and (d) SD = 5.0. The solid line is the original data and the dotted lines are the SSS data.
We found that our method can identify RW data, even when the SD of GRN used as the disturbance term changes and when additional irregular fluctuations (or observational noise) are not dominant. Hence, we conclude that our idea can investigate whether data can be approximated by RW. In the case when additional observational noise is dominant, the RW conclusion is correctly rejected. In the next section, we apply our proposed method to data generated by a SV model.

5.2. The case of using the Heston SV model

It is known that RW can be approximated by stochastic differential equations. To generate RW data using a different model and to investigate how our method works, we use the Heston SV model, in which the
Fig. 6. (Color online) A plot of the AC and the AMI for $S_t$ data of the Heston SV model, where the original data is used in (a) and (b), and the difference data is used in (c) and (d). The solid line is the original data of each case and the dotted lines are the SSS data.

Fig. 7. Stock market data examined in this paper: (a) and (b) the daily closing price of S&P500, and (c) and (d) the daily closing price of Nikkei225. Both data are from 3 January 1950 to 30 December 2005. (b) and (d) show the first difference data of (a) and (c). The horizontal axis shows the trading day in (a)-(d) and the ordinate axis shows the price in (a) and (c).
Fig. 8. (Color online) A plot of the AC and the AMI for S&P500 and Nikkei225 data, where the original data is used in (a)–(d), and the difference data is used in (e)–(h). (a), (b), (e) and (f) are S&P500 data, and (c), (d), (g) and (h) are Nikkei225. The solid line is the original data of each case and the dotted lines are the SSS data.
randomness of the variance process varies as the square root of variance \[24\]. The Heston SV model is one of the common SV models \[25\].

In Section 5.1, we confirmed that when additional irregular fluctuations (or observational noise) are not dominant, our proposed method works well. The case is not a particular for the example but ubiquitous. Hence, in this section, to investigate availability of the proposed method, we simply use noise free data of the Heston SV model.

The model can be given by

\[
\begin{align*}
dS_t &= \mu S_t dt + \sqrt{V_t} S_t dW^1_t, \\
dV_t &= \kappa(\theta - V_t) dt + \sigma \sqrt{V_t} dW^2_t, \\
dW^1_t dW^2_t &= \rho dt,
\end{align*}
\]

(2)

where \(S_t\) is the price, \(V_t\) is the variance, and \(dW^1_t\) and \(dW^2_t\) are increments of Brownian motion with correlation \(\rho \in [-1, 1]\). See more details elsewhere \[26–28, Chapter 7\]. We use \(\rho = -0.05\), \(\mu = 0.02\), \(\sigma = 0.3\), \(\theta = 0.04\) and \(\kappa = 2.0\) \[28, p. 163\]. Using the Euler scheme with the sampling interval 0.01 and the Cholesky decomposition, we generate the data and take the price data \(S_t\) as observational data.

Fig. 5(a) shows irregular fluctuations and some trends, and Fig. 5(b) shows that the SD of the difference data is not constant, increases and decrease like actual financial data, for example Figs. 7(b) and (d).

As done in the previous section, we first apply the SSS method to the original data. As shown in Figs. 6(a) and (b), both the AC and the AMI of the original data fall outside the distribution of the SSS data. Hence, we reject the NH. Then, we apply the SSS method again to the first difference data. As shown in Figs. 6(c) and (d), both the AC and the AMI of the difference data fall within the distribution of the SSS data. Hence, we cannot reject the NH. From this result we expect that the price data \(S_t\) is a RW.
6. Applications

Based on the results of our computational studies, we apply the proposed method to financial data in different markets: (1) daily stock market: Standard & Poor’s 500 (S&P500) in US market and Nikkei225 in Japanese market, (2) daily exchange rate: British Pound/US dollar (GBP/USD) and Japanese Yen/US dollar (JPY/USD), and (3) commodity market: daily gold price and daily crude oil price. The behaviors of all data show irregular fluctuations and long-term trends (Figs. 7(a), (c), 10(a), (c), 12(a), and (c)), and the first difference data show that the volatility of the data are not constant and clearly increase and decrease over time (Figs. 7(b), (d), 10(b), (d), 12(b), and (d)). From these behaviors we expect that these data are RW composed of time-varying random variables. In all cases, we use $A = 1.0$ and generate 99 SSS data.$^5$

6.1. Daily stock market

In this section we examine two stock market data, the daily closing price data of S&P500 and Nikkei225, where both data are from 3 January 1950 to 30 December 2005. Fig. 7 shows the original and the first difference data.

We first apply the SSS method to the original data. When the data is S&P500, Fig. 8(a) shows that the difference between the AC of the original data and the distributions of SSS data is arguably small, and Fig. 8(b) shows that the AMI of the original data fall outside the distributions of the SSS data. Also, when the data is Nikkei225, Figs. 8(c) and (d) show that both the AC and the AMI of the original data fall outside the distributions of the SSS data. Hence, we conclude that the irregular fluctuations of S&P500 and Nikkei225 have some kind of dynamics. This result indicates that there is a possibility that the data are RW at this stage. Then, we apply the SSS method again to the first difference data. Figs. 8(e)–(h) show that the AC and the AMI

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$^5$We generate another four set of 99 SSS data for all financial data (that is to say, we repeat each test five times). We show the typical result in each case.
Fig. 11. (Color online) A plot of the AC and the AMI for daily GBP/USD and JPY/USD exchange rate data, where the original data is used in (a)–(d), and the difference data is used in (e)–(h). (a), (b), (e) and (f) are GBP/USD data, and (c), (d), (g) and (h) are JPY/USD data. The solid line is the original data of each case and the dotted lines are the SSS data.
fall within the distribution of the SSS data. The result indicates that both the first difference data are time-varying random variables (ID random variables). Hence, we conclude that the daily closing price data of S&P500 and Nikkei225 are essentially RW.

Here, to indicate how the SSS method destroys local structures and preserves the global behaviors (trends) in detail, we show one of the SSS data of S&P500 data and the first difference data. Figs. 9(a) and (c) show very similar behavior to Figs. 7(a), and (b), and 9(b), and (d) show that local structures are different between the two.

6.2. Daily exchange rate

In this section we examine two exchange rate data, the daily GBP/USD and JPY/USD exchange rate data, where both data are from 2 January 1974 to 30 December 2005. Fig. 10 shows the original and the first difference data.

We first apply the SSS method to the original data. Although Figs. 11(a) and (c) show that the difference between the AC of the original data and the distributions of SSS data is not large, Figs. 11(b) and (d) show that the AMI of the original data fall outside the distributions of the SSS data. Hence, we conclude that the irregular fluctuations of GBP/USD and JPY/USD data have some kind of dynamics. Then, we apply the SSS method again to the first difference data. Figs. 11(e)–(h) show that the AC and the AMI fall within the distribution of the SSS data. The result indicates that both the first difference data are time-varying random variables. Hence, we conclude that the daily GBP/USD and JPY/USD exchange rate data are essentially RW.

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6We found that although the AMI of S&P500 at time lag 5 and the AC of Nikkei225 at time lag 2 sometimes fall outside the distribution of the SSS data, the difference is not significant and the AMI and the AC at other time lags clearly fall within the distribution. Hence, we still conclude that we fail to reject the NH in these cases.
Fig. 13. (Color online) A plot of the AC and the AMI for the daily gold price and crude oil price data, where the original data is used in (a)–(d), and the difference data is used in (e)–(h). (a), (b), (e) and (f) are the gold price data, and (c), (d), (g) and (h) are the crude oil price data. The solid line is the original data of each case and the dotted lines are the SSS data.
6.3. Commodity market

In this section we examine two commodity market data, the daily gold price data from 5 January 1970 to 30 December 2005 and the daily crude oil price data at New York Mercantile Exchange (NYMEX) futures from 2 January 1986 to 30 December 2005. Fig. 12 shows the original and the first difference data.

As well as two examples examined previously, when we apply the SSS method to the original data, the difference between the AC of the original data and the distributions of SSS data is small, and the AMI of the original data fall outside the distributions of the SSS data. See Figs. 13(a)–(d). Hence, we conclude that the irregular fluctuations of gold price and crude oil price data have some kind of dynamics. Then, we apply the SSS method again to the first difference data. Figs. 13(e)–(h) show that the AC and the AMI fall within the distribution of the SSS data. The result indicates that both the first difference data are time-varying random variables. Hence, we conclude that the daily gold price and crude oil price data are essentially RW.

7. Conclusion

We have described an algorithm for investigating whether data can be approximated by a RW. Our method appears particularly well suited to identify nontrivial dynamics (not a RW) in noisy and nonstationary time series, when additional observational noise are not large. The advantage of applying our method is that the method does not depend on the specific data distribution, although previously proposed methods do. Applying our method finds that all financial data examined in this paper are RW whose first differences are ID random variables or time-varying random variables.

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References