\[ x^2 - y_1^2 + y_2^2 = z^2 \]

Define the reduced cost vector by
\[
q_i = \begin{cases} 0 & \text{if } i \in B \\ \text{else} & \end{cases}
\]
and \( q'A = q \).

Let \( A \) be a matrix consisting of \( m \) linearly independent columns
\[ 0 \preceq x \]
\[ q = A^\top x \]
such that
\[ x_0^2 = z \]
with

Consider a matrix representation of the simplex method.

Before answering these questions we first look at
\[ \begin{array}{c} x^0 + q^0 \rightarrow q \\ c \end{array} \]
changed slightly, i.e.

2. The change in the optimal solution if the requirement vector \( b \) is
\[ c \leftrightarrow c + \epsilon \]
changed slightly, i.e.

1. The change in the optimal solution if the cost coefficient vector
Suppose we are given a particular LP problem and the problem has

Sensitivity Analysis

\[(I - I) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 \end{pmatrix}
\]
The dual solution is
\[ \begin{array}{c} z = 0 \\ 0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{array} \]
All \( x \) \ not in the optimal solution is obtained which is

\[ \begin{array}{llllll} & & & & & \\ 1 & 1 & 1 & 0 & 0 & \end{array} \]
\[ \begin{array}{llllll} & & & & & \\ 3 & 4 & 5 & 0 & 0 & \end{array} \]

New tableau:

\[ \begin{array}{llllll} & & & & & \\ 4 & 5 & 3 & 0 & 0 & \end{array} \]

So, \( y = 3, q \) is in the basis.
Note that if \( r = 1 \) is a normal choice, but we can also choose other \( r \).

\[
\begin{bmatrix}
1 & 0 & 2 \\
0 & 1 & 2 \\
0 & 0 & 1
\end{bmatrix} = [a_0 \ b_0 \ c_0] = \mathbf{g}
\]

as long as \( z' = \mathbf{z} > 0 \). For example, we may choose \( r = 2 \). Then

\[
\begin{array}{c|c|c}
0 & \begin{bmatrix} 0 & 0 & 3 & 1 & \mathbf{c} \end{bmatrix} & \mathbf{f}_d - \mathbf{z} \\
9 & 0 & 0 \\
5 & V = V_1 - \mathbf{B} & 0 \\
\hline
9 & [0 \ 0 \ 0 \ 3 - 1 - 3 -] = \mathbf{p} & \mathbf{a} \mathbf{x} \\
\end{array}
\]

is feasible. The tableau is

\[
\begin{bmatrix}
9 \\
5 \\
4
\end{bmatrix} = \begin{bmatrix} 0 \\
q_1 - \mathbf{B} \end{bmatrix} = \begin{bmatrix} 0 \\
\mathbf{a} \mathbf{x}
\end{bmatrix}
\]

and

\[
\mathbf{g} = \mathbf{1} - \mathbf{B} \cdot \mathbf{x} = \mathbf{0} \mathbf{x} = \begin{bmatrix} 9 \ 5 \ 4 \end{bmatrix} \mathbf{x} = \mathbf{x}
\]

Choose \( \mathbf{B} = \mathbf{1} - \mathbf{B} \cdot \mathbf{x} = \mathbf{0} \mathbf{x} \)

\[
\begin{bmatrix}
9 \\
5 \\
4
\end{bmatrix} \geq \mathbf{x}
\]

\[
\begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 0 & 3 & 1 \\
0 & 0 & 1 & 1
\end{bmatrix} \geq \mathbf{x}
\]

The standard form is

\[
\begin{bmatrix}
0 \\
9 \\
5 \\
5 \\
4
\end{bmatrix} \geq \mathbf{x}
\]

\[
\begin{bmatrix}
0 & 1 & 2 \\
1 & 1 & 2 \\
0 & 1 & 1
\end{bmatrix} \geq \mathbf{x}
\]

\[
\begin{bmatrix}
5 \\
4
\end{bmatrix} \geq \mathbf{x}
\]

\[
\begin{bmatrix}
0 & 1 & 2 \\
1 & 1 & 2 \\
0 & 1 & 1
\end{bmatrix} \geq \mathbf{x}
\]

\[
\begin{bmatrix}
5 \\
4
\end{bmatrix} \geq \mathbf{x}
\]

Example: Solve the following LP problem.

\[
\begin{array}{c|cc|c}
\mathbf{a}_d & \mathbf{g} - V_1 - \mathbf{B}_d & \mathbf{z} \\
-5 & 0 & 0 \\
9 & 9 \\
10 & 10 \\
\hline
9 & \mathbf{g}_d & \mathbf{a} \mathbf{x} \\
\end{array}
\]

The tableau in the (Preliminary) Simplex method can be expressed as

\[
\mathbf{N}_1 - \mathbf{B}_d \mathbf{g} \geq \mathbf{N}_d
\]

If \( \mathbf{g} \) is an optimal basis, then \( \mathbf{N}_1 \geq \mathbf{N}_d \) (comparative condition). This

\[
\begin{bmatrix}
\mathbf{a}_d - \mathbf{V}_1 - \mathbf{B}_d & -1 & \mathbf{N}_d - \mathbf{N}_1 \mathbf{g} \\
\end{bmatrix} = \mathbf{N}_d - \mathbf{N}_1 \mathbf{g} \geq \mathbf{N}_d
\]

\[
\mathbf{0} = \mathbf{N}_d - \mathbf{B}_d \mathbf{g} = \mathbf{a} \mathbf{x}
\]

If \( \mathbf{a}_d \) is an

\[
\begin{bmatrix}
\mathbf{a}_d \\
\end{bmatrix} = \mathbf{g}
\]

then

\[
\begin{bmatrix}
\mathbf{a}_d \\
\end{bmatrix} = \mathbf{g}
\]

and
Consider the optimal tableau for the matrix form:

\[
\begin{bmatrix}
1 & 0 & -1 & 0 & 0 & \overline{z} \\
0 & 5/2 & 1/2 & 1 & 0 & 0 \\
0 & 5/2 & 1/2 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & \overline{z} \\
\end{bmatrix}
\]

Changes in the cost coefficients:

- \(z \to z + 2\)
- \(x \to x + 2\)

The optimal solution is satisfied and so an optimal solution is

\[
\begin{bmatrix}
1 & 0 & -1 & 0 & 0 & \overline{z} \\
0 & 5/2 & 1/2 & 1 & 0 & 0 \\
0 & 5/2 & 1/2 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & \overline{z} \\
\end{bmatrix}
\]

So.

\[
\begin{bmatrix}
1 & 0 & -1 & 0 & 0 & \overline{z} \\
0 & 5/2 & 1/2 & 1 & 0 & 0 \\
0 & 5/2 & 1/2 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & \overline{z} \\
\end{bmatrix} = \mathbf{I} - \mathbf{A}
\]

New tableau:

\[
\begin{bmatrix}
1 & 0 & -1 & 0 & 0 & \overline{z} \\
0 & 5/2 & 1/2 & 1 & 0 & 0 \\
0 & 5/2 & 1/2 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & \overline{z} \\
\end{bmatrix}
\]

Therefore,

\[
\begin{bmatrix}
1 & 0 & -1 & 0 & 0 & \overline{z} \\
0 & 5/2 & 1/2 & 1 & 0 & 0 \\
0 & 5/2 & 1/2 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & \overline{z} \\
\end{bmatrix}
\]

The next tableau is

\[
\begin{bmatrix}
1 & 0 & -1 & 0 & 0 & \overline{z} \\
0 & 5/2 & 1/2 & 1 & 0 & 0 \\
0 & 5/2 & 1/2 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & \overline{z} \\
\end{bmatrix}
\]

and so.
Therefore, \( z = \left( \frac{1}{2} \right) (0 \ 1 \ 0 \ 0) = 0.5 x_1 + 0.5 x_2 \)

\[ (z, x_1, x_2, x_3, x_4) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \]

The optimal solution is still optimal.

So, if the problem is still optimal, and the old optimal solution is still optimal, and the optimal basis unchanged, we may continue.

\[ (0 \ 0 \ 1 \ 0 \ 0) - \frac{1}{2} (0 \ 0 \ 1 \ 0 \ 0) = \frac{1}{2} (0 \ 0 \ 1 \ 0 \ 0) \]

The standard form for the problem is

\[
\begin{align*}
0 & \leq x_1 + x_2 \\
0 & \leq x_1 - x_2 \\
x_1 & = x_2 \\
\text{min} & \quad z = x_1 - x_2
\end{align*}
\]

Example: Consider the following LP problem:

\[
\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - x_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} x_1
\]

where

\[
0 \geq (2x_1 + x_2 - x_3 - x_4 = x_1 + x_2 + x_3 + x_4 \geq \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} x_1
\]

So, optimal \( z \) changes only if \( z \) becomes non-optimal basis.
From this we have that

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The corresponding tableau is

\[
\begin{align*}
0 &= 0 - (1) (0) = \\
\bar{x}_y &= 0 - (1) (0) = \\
\bar{x}_y &= 0 - (0) (0) = \\
\bar{x}_y &= (1) (0) = \\
(1) (1) (0) &= \\
\end{align*}
\]

is given by

\[
g_0 - 4g - 1 + x - z = 0.
\]

Let \( g = (0, 0, 0) \). Then \( g^T = (0, 0, 0) \). The change in \( t \)

Find out how much can \( b \) be altered before the optimal basis
cases to be optimal.

Obviously, \( -5 - 6g \leq 0 \). Thus, \( g \) is \( \leq 5 \). In this case,

<table>
<thead>
<tr>
<th>x</th>
<th>z</th>
<th>( g_0 - \bar{t}_x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

So, the tableau becomes

\[
\begin{align*}
0 &= 0 - (1) (0) = \\
\bar{x}_y &= 0 - (1) (0) = \\
\bar{x}_y &= 0 - (0) (0) = \\
\bar{x}_y &= (1) (0) = \\
(1) (1) (0) &= \\
\end{align*}
\]

Notes:

(i) Change in \( a_0 \) does not affect the new optimal cost since a

mon-basic coefficient.

(ii) Change in \( a_{ij} \) is a non-basic variable (called it a

non-basic coefficient)
Example: Consider the LP problem

is independent of \( q \) and \( q' \), and thus the solution remains

\[ 0 < (q + q') (1 - \beta) = \bar{a} x + \bar{a} x \]

The optimal condition \( \beta \) in order to maintain feasibility.

So an optimal solution is obtained with

\[ \begin{array}{c|cccc|c}
   \beta & 0 & 0 & 3/16 & 1/16 \\
   \hline
   3/16 & 1 & 3/16 & 1/16 & 0 \\
   3/16 & 0 & 3/16 & 1/16 & 1 \\
   \hline
   0 & 0 & 1 & 1 & \beta \end{array} \]

Clearly, if \( \bar{a} \) is no longer optimal, then we continue with the

\[ \begin{array}{c|cccc|c}
   x & 0 & 1 & 0 & 4 \\
   \hline
   3/16 & 0 & 1 & 3/16 & 0 \\
   3/16 & 0 & 1 & 3/16 & 1 \\
   0 & 0 & 1 & \beta & \end{array} \]

So if \( \bar{a} \) is no longer optimal, the corresponding tableau becomes

\[ \frac{\bar{a}}{\bar{a}} \rightarrow \frac{\alpha}{\alpha} \]

and the basic vector is still \( (1, 1/2) \). Therefore, we continue to solve these cases.

Consider the final optimal tableau in the primal simplex method:
are listed below.

The number of hours for the three operations and the profits
Examine a company makes three models of air-conditioners, A, B, and C. The numbers of hours for the three operations and the profits.

The feasible solution we have to perform one step of the dual simplex
The solution is optimal but not feasible. To obtain an optimal
So the new tableau is

$$
\begin{align*}
\begin{array}{ccc|c}
4 & 0 & 0 & -z_4 - z_2 \\
5 & 1 & 1 & 0 + x_2 \\
0 & 1 & 1 & -z_4 - z_2 \\
0 & 1 & 1 & 0 + x_2 \\
\end{array}
\end{align*}
$$

from which we have

$$
\begin{align*}
\begin{bmatrix}
4 \\
5 \\
0 \\
0 \\
\end{bmatrix}
&= \text{new } A
\\
\begin{bmatrix}
0 \\
1 \\
1 \\
1 \\
\end{bmatrix}
&= \text{new } x
\end{align*}
$$

when \( \frac{y_1}{x} = 0 \), we have that \( -z_4 \geq 0 \). When \( \frac{y_1}{x} \neq 0 \), we have that \( -z_4 \geq 0 \).
(c) What is the profit increase if the baking time is increased by one hour?

If it becomes worthwhile to produce:

\[
\begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix} + \begin{pmatrix}
-320 - 6g_1 < 0 \\
-640 - g_1 > 0
\end{pmatrix}
\]

So the profit of \( A \) must be increased by more than \( 5350 \) before

The primal shadow prices for \( x_1 \) and \( x_2 \):

\[
\begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix}
\]

This is not used.

Let \( g_1 = 0 \) and so

\[
\begin{pmatrix}
-320 - 6g_1 < 0 \\
-640 - g_1 > 0
\end{pmatrix}
\]

2. \( x_2 = 50 \text{ hours} \) means that there is 50 hours of assembly time which from the results we see that it is not worth to produce. 

Hence:

\[
\begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix}
\]

So an optimal feasible solution is obtained. 

The primal shadow method: Initial tableau

The primal shadow method: Initial tableau

The primal shadow method: Initial tableau

The primal shadow method: Initial tableau

The primal shadow method: Initial tableau
\[ \begin{pmatrix} \frac{1}{4} \varepsilon - \frac{1}{2} \gamma \varepsilon - \frac{1}{2} \gamma \varepsilon \end{pmatrix} = \gamma \varepsilon 
\]

In general, \( \gamma \varepsilon \geq 0 \) then \( \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \)

10. Profit will be increased by \( \varepsilon \).

So, \( \varepsilon \) can vary as much as 100 while the solution remains optimal.

\[
0 \leq \left( \frac{1}{4} \varepsilon - \frac{1}{2} \gamma \varepsilon \right) \leq 100.
\]

This is

\[
0 \leq \left( \frac{1}{4} \varepsilon - \frac{1}{2} \gamma \varepsilon + 150 \right) = \varepsilon y + \varepsilon x
\]

From this we have

Check the feasibility.