3. For the constraints in (i) we have
\[ x_1 \geq 10 - My_1, \quad x_2 \geq 10 - M(1 - y_1), \] (1)
where \( M \gg 1 \) is a constant and \( y_1 = 0 \) or 1.
For (ii),
\[ 2x_1 + x_2 \geq 15 - My_2, \quad x_1 + x_2 \geq 15 - My_3, \] (2)
\[ x_1 + 2x_2 \geq 15 - My_4, \] (3)
where \( y_2, y_3, y_4 \) satisfy
\[ y_2 + y_3 + y_4 \leq 2, \quad y_2, y_3, y_4 \in \{0, 1\}. \] (4)

Note that \(|x_1 - x_2| = 0, 5 \) or 10 are equivalent to \( x_1 - x_2 = \pm 5 \) or \( \pm 10 \). So, we introduce \( y_i \) for \( i = 5, 6, \ldots, 9 \) and replace the constraints by
\[ x_1 - x_2 = y_5 + 5y_6 - 5y_7 + 10y_8 - 10y_9 \] (5)
\[ y_5 + y_6 + \cdots + y_9 = 1, \quad y_i \in \{0, 1\} \] (6)
for \( i = 5, 6, \ldots, 9 \). Therefore, the problem becomes
\[ \text{min } z = f_1(x_1) + f_2(x_2) \text{ subject to (1)–(6) and } x_1, x_2 \geq 0. \]

Q4. Let \( x_{ik} \) be the number of units from factory \( i \) to warehouse \( k \) and \( y_{kj} \) be the number of units from warehouse \( k \) to retailer \( j \). Then the total cost is
\[ z = \sum_{i=1}^{m} \sum_{k=1}^{p} c_{ik}x_{ik} + \sum_{k=1}^{m} \sum_{j=1}^{n} d_{kj}y_{kj} + \sum_{k=1}^{m} f_ku_k, \] (7a)
where \( u_k = 1 \) if the \( k \)th warehouse is used or 0 otherwise. The IP then becomes
\[ \text{min } z \]
\[ \text{s.t. } \sum_{i=1}^{n} x_{ik} \leq q_k, \quad k = 1, 2, \ldots, p, \]
\[ \sum_{k=1}^{m} x_{ik} \leq a_i, \quad i = 1, 2, \ldots, m, \]
\[ \sum_{k=1}^{m} y_{kj} \geq b_j, \quad j = 1, 2, \ldots, n, \]
\[ \sum_{j=1}^{n} y_{kj} \leq q_k, \quad k = 1, 2, \ldots, p, \]
\[ \sum_{i=1}^{m} x_{ik} \leq q_ky_k, \quad k = 1, 2, \ldots, p, \]
\[ x_{ik}, y_{kj} \text{ are non-negative integers, } u_k \in \{0, 1\} \] (7b)

5. Let \( x_i = 1 \) if \( i \)th project is chosen or 0 otherwise for \( i = 1, 2, 3, 4, 5 \). Then, the problem is
\[ \text{max } x_1 + 1.8x_2 + 1.6x_3 + 0.8x_4 + 1.4x_5 \]
s.t. \[ 6x_1 + 12x_2 + 10x_3 + 4x_4 + 8x_5 \leq 20, \]
\[ x_i \in \{0, 1\}, \quad i = 1, 2, \ldots, 5. \]

6. (a) From the constraints we see that \( x_i = 0, 1, 2, \) or 3 for \( i = 1, 2 \) and \( x_1 + x_2 \leq 3 \). Therefore, let \( x_i = \sum_{j=1}^{3} y_{ij} \) for \( i = 1, 2 \). The problem can be formulated as
\[ \text{max } z = z \left( \sum_{j=1}^{3} y_{1j}, \sum_{j=1}^{3} y_{2j} \right) \]
s.t. \[ \sum_{j=1}^{3} y_{ij} \leq 1, \quad i = 1, 2, \]
\[ \sum_{j=1}^{3} y_{1j} + \sum_{j=1}^{3} y_{2j} \leq 3, \]
\[ y_{ij} \in \{0, 1\}, \quad i = 1, 2, \quad j = 1, 2, 3. \] (b) Let \( x_i = \sum_{j=1}^{3} y_{ij} \) for \( i = 1, 2 \). Then we have
\[ \text{max } z = z \left( \sum_{j=1}^{3} y_{1j}, \sum_{j=1}^{3} y_{2j} \right) \]
sub. to \[ \sum_{j=1}^{3} y_{1j} + \sum_{j=1}^{3} y_{2j} \leq 3, \]
\[ y_{ij} \in \{0, 1\}, \quad i = 1, 2, \quad j = 1, 2, 3. \] (8)

A solution to the LP relaxation of the problem is \( z = 6.5 \) and \( x = (2/3, 1, 1, 1, 1) \). (Note you have to assume \( x_i \leq 1 \) for \( i = 1, \ldots, 5 \). Otherwise, there is no bounded solution by lp-solve.) Therefore, the bound on the optimal \( z \) is 6.

Since \( x_1 \) is non-integer, we branch on it.

When \( x_1 = 0 \), we have the subproblem \( S_0 \)
\[ \text{max } -x_2 + 5x_3 - 3x_4 + 4x_5 \]
s.t. \[ -x_2 + 7x_3 - 5x_4 + 4x_5 \leq 6, \]
\[ x_2 + 2x_3 - 4x_4 + 2x_5 \leq 0; \]
\[ x_i \in \{0, 1\}. \]

Using lp-solve we get solution to the LP relaxation as \( z^* = 6 \) and \( x = (0, 0, 1, 1, 1) \). This is an integer solution and thus this branch is fathomed. This is our current incumbent solution.

When \( x_1 = 1 \), we have the subproblem \( S_1 \)
\[ \text{max } 2 - x_2 + 5x_3 - 3x_4 + 4x_5 \]
s.t. \[ -x_2 + 7x_3 - 5x_4 + 4x_5 \leq 3, \]
\[ x_2 + 2x_3 - 4x_4 + 2x_5 \leq -1; \]
\[ x_i \in \{0, 1\}. \]
Using lp-solve we get \( z = 6.28571 \) and \( x = (1, 1, 0.857143, 1, 1) \). This is a non-integer solution. But the bound is 6 which is equal to the incumbent solution. Therefore, this branch can also be fathomed.

So, the optimal solution is \( z^* = 6 \) and \( x^* = (0, 0, 1, 1, 1) \).

10. Let \( x_i \) be the number of product \( i \) and \( y_i = 1 \) if product \( i \) is chosen or 0 otherwise, for \( i = 1, 2, 3, 4 \). Then the problem can be formulated as (scaled by 1000)

\[
\begin{align*}
\text{max} & \quad 70x_1 + 60x_2 + 90x_3 + 80x_4 \\
& \quad + 50y_1 + 40y_2 + 70y_3 + 60y_4 \\
\text{s.t.} & \quad \sum_{j=1}^{4} y_i \leq 2, \\
& \quad y_3 + y_4 \leq y_1 + y_2, \\
& \quad 5x_1 + 3x_2 + 6x_3 + 4x_4 \leq 6000 + My_5, \\
& \quad 4x_1 + 6x_2 + 3x_3 + 5x_4 \leq 6000 + My_6, \\
& \quad y_5 + y_6 = 1, \\
& \quad x_i \text{are non-negative integers}, \quad y_j \in \{0, 1\}, \\
& \quad i = 1, 2, 3, 4; j = 1, \ldots, 6,
\end{align*}
\]

where \( M \gg 1 \) is a constant.