1. Find all of the minimal spanning trees for the following network.

![Network Diagram](image)

2. Find a minimal spanning tree for the network shown, both by Krushal's and Prim's algorithms.

![Network Diagram](image)

Figure 1

3. Consider the undirected network depicted in Figure 1, where numbers along the arcs represent distances between nodes. Assume that the distance from $i$ to $j$ is the same as that from $j$ to $i$ (i.e. all arcs are two-way streets). The problem is to determine the shortest path and the length of the shortest path from node 1 to node 6.

4. Give an example of a network where there are some arcs with the same cost, but the minimal spanning tree is unique.

5. Explain why a graph $G$ remains connected even after deleting an arc $a$ if and only if $a$ belongs to some cycle in $G$.

6. The planned locations of Ethernet outlets to be installed in the Arts Faculty are given in the figure, where numbers on the arcs represent costs in thousands of dollars. Since the cost of laying the fibre-optic cable is much greater than the running costs, and since the cable has enough capacity that one only requires a path between every computer and the route at R, the Faculty would like to install only those
cable segments needed to ensure that every outlet has a path to the route. Find the best solution for the Faculty.

7. Draw networks whose node-arc incidence matrices are the following:

   (a) \[
   \begin{pmatrix}
   1 & -1 \\
   -1 & 1
   \end{pmatrix}
   \]  
   (b) \[
   \begin{pmatrix}
   1 & 0 & -1 & -1 & 0 \\
   -1 & 1 & 0 & 1 & -1 \\
   0 & -1 & 1 & 0 & 1
   \end{pmatrix}
   \]  
   (c) \[
   \begin{pmatrix}
   1 & 1 & 1 & 0 & 0 & 0 \\
   -1 & 0 & 0 & 1 & 1 & 0 \\
   0 & -1 & 0 & -1 & 0 & 1 \\
   0 & 0 & -1 & 0 & -1 & -1
   \end{pmatrix}
   \]  
   (d) \[
   \begin{pmatrix}
   -1 & 0 & 0 & 1 & 0 & 0 \\
   0 & 1 & -1 & 0 & -1 & 0 \\
   0 & 0 & 1 & 0 & 0 & 1 \\
   0 & 0 & 0 & 0 & 0 & 0 \\
   1 & 0 & 0 & -1 & 0 & 0 \\
   0 & -1 & 0 & 0 & 1 & 1
   \end{pmatrix}
   \]

8. Use the tree algorithm to find a path from 1 to 10 in the network below.

9. Find the maximum flow from A to B in the following network. Arcs have capacity as shown and are bidirectional.

10. Consider the following directed network where s is the source node, n is the sink node, and the numbers along the arcs denote the capacities of flows
(a) Illustrate the following notations with an example for the above network:
   i. a path connecting the source and the sink,
   ii. a cut separating the source and the sink,
   iii. the capacity of a cut.

(b) Find the maximum flow from the source to the sink using the labelling method.
(c) Find the minimum cut, and verify the max-flow min-cut theorem.

11. Consider a street network as shown below

```
    s      1      3      n
   / \
 30  10  20  10  70
(8)  80 200 100 40
```

   The numbers on the arcs represent the traffic flow capacities. The problem is to place one-way signs on streets not already oriented so as to maximize the traffic flow from the point s to the point n.

12. Determine the maximal flow through the network in the following figure.

```
source       3       5       sink
    \
   / \
1  7  1
```

13. Consider the network in the following figure.
Numbers along arcs are capacities of the flow. Notice that the network is undirected. The problem is to determine the maximal flow between node 1 and node 7, assuming infinite availability at node 1.