1. Find all the basic solutions of
\[
\begin{align*}
  x_1 + x_2 + 2x_3 &= 1 \\
  3x_1 - x_2 + 2x_3 &= 2.
\end{align*}
\]
Which of them are feasible if \( x \geq 0 \) is required?

2. Determine the optimal solution for the following LP by enumerating all the basic solutions.
\[
\begin{align*}
  \text{minimize} \quad & z = x_1 + 2x_2 - 3x_3 - 2x_4 \\
  \text{subject to} \quad & x_1 + 2x_2 - 3x_3 + x_4 = 4 \\
  & x_1 + x_2 + x_3 + 2x_4 = 4 \\
  & x_1, x_2, x_3, x_4 \geq 0.
\end{align*}
\]

3. Solve the following problem by the simplex method
   
   (a)
   \[
   \begin{align*}
   & \text{maximize} \quad z = 120x_1 + 100x_2 \\
   & \text{subject to} \quad x_1 + x_2 \leq 4 \\
   & \quad 5x_1 + 3x_2 \leq 15 \\
   & \quad x_1, x_2 \geq 0.
   \end{align*}
   \]

   (b)
   \[
   \begin{align*}
   & \text{maximize} \quad z = 6x_1 + 7x_2 \\
   & \text{subject to} \quad 2x_1 + 3x_2 \leq 400 \\
   & \quad x_1 + x_2 \leq 150 \\
   & \quad x_1, x_2 \geq 0.
   \end{align*}
   \]

   (c)
   \[
   \begin{align*}
   & \text{maximize} \quad z = 10x_1 + 12x_2 + 15x_3 \\
   & \text{subject to} \quad x_1 - 2x_2 \leq 6 \\
   & \quad 3x_1 + x_3 \leq 9 \\
   & \quad x_2 + 3x_3 \leq 12 \\
   & \quad x_1, x_2, x_3 \geq 0.
   \end{align*}
   \]
(d)\[
\begin{align*}
\text{min} & \quad z = -x_1 - x_2 \\
\text{subject to} & \quad x_1 + 4x_2 \leq 14 \\
& \quad x_1 - x_2 \leq 3 \\
& \quad x_1 + x_2 \leq 4 \\
& \quad x_1, x_2 \geq 0.
\end{align*}
\]

(e)\[
\begin{align*}
\text{max} & \quad z = 2x_1 + x_2 \\
\text{subject to} & \quad x_1 + 4x_2 \geq 14 \\
& \quad x_1 - x_2 \leq 3 \\
& \quad x_1 + x_2 \geq 10 \\
& \quad x_1, x_2 \geq 0.
\end{align*}
\]

(f)\[
\begin{align*}
\text{min} & \quad z = x_1 + 3x_2 \\
\text{subject to} & \quad x_1 + 4x_2 \leq 14 \\
& \quad x_1 - x_2 \leq 3 \\
& \quad x_1 + x_2 \geq 10 \\
& \quad x_1, x_2 \geq 0.
\end{align*}
\]

(g) \text{min} \ z = x_1 + 3x_2 \text{ subject to the constraints in item (e)}

4. Consider the following problem:

\[
\begin{align*}
\text{maximize} & \quad z = x_2 \\
\text{subject to} & \quad 2x_1 + 3x_2 \leq 9 \\
& \quad |x_1 - 2| \geq 1 \\
& \quad x_1, x_2 \geq 0.
\end{align*}
\]

Illustrate the constraint set graphically and find the optimal solution. How might the simplex method be employed to solve such a problem?

5. Use the two phase method to solve the following LP problem:

\[
\begin{align*}
\text{maximize} & \quad z = 3x_1 + 2x_2 \\
\text{subject to} & \quad 2x_1 + x_2 \leq 2 \\
& \quad 3x_1 + 4x_2 \geq 12 \\
& \quad x_1, x_2 \geq 0.
\end{align*}
\]
6. Two products, A and B, are made involving two chemical operations for each. Each unit of product A requires 2 hours on Operation 1 and 3 hours on Operation 2. Each unit of product B requires 3 hours on Operation 1 and 4 hours on Operation 2. Available time for Operation 1 is 16 hours and for Operation 2 is 24 hours.

Product A sells for $4 profit per unit, while B sells for $10 profit per unit. By-product C can be sold at a unit profit of $3, but if it cannot be sold, the destruction cost is $2 per unit. Forecasts show that up to 5 units of C can be sold. The company gets 2 units of C for each unit of B produced.

The problem is to determine the production quantity of A and B, keeping C in mind, so as to make the largest profit. Formulate as an LP problem and solve by the simplex algorithm.

7. Solve the following problem

$$\begin{align*}
\text{min} & \quad z = 3x_3 + x_4 + 2x_5 \\
\text{subject to} & \quad x_3 + x_4 - 2x_5 - x_1 = 1 \\
& \quad x_3 - 2x_4 + x_5 - x_2 = -1 \\
& \quad x_i \geq 0, \quad i = 1, \ldots, 5.
\end{align*}$$

8. A manufacturer of metal products makes three products A, B and C, all of which require machining, polishing and assembling of component parts. The amounts of these operations required for one unit of A are 3, 1 and 2 hours, respectively. Similarly, they are 2, 1 and 1 hours for product B and 4, 1 and 2 hours for product C. There are 300 hours of total machining time available, 100 hours of polishing time and 200 hours of assembling time. The unit profits that can be made from sale of products A, B and C are $3, $2 and $4 respectively. How many units of each product should be produced so as to maximize total profit? Is the optimal basic feasible solution unique?

9. Is it possible for a linear program to have more than one optimal \(x\) value yet only a finite number of optimal \(x\) values? (Hint: Look at a simple 2D example, and/or recall that the set of feasible solutions is convex.)