1. Solve the following problem, starting from $x^0 = (1, 1)^T$, by Newton’s method so that the error is less than $10^{-8}$: min $\left( x_1^2 + x_1^2 x_2^2 + 3x_2^4 \right)$.

2. Consider the solution of following linear system

$$
\begin{align*}
x_1 - 2x_2 + 3x_3 &= 2, \\
3x_1 - 2x_2 + x_3 &= 7, \\
x_1 + x_2 - x_3 &= 1
\end{align*}
$$

by an unconstrained optimisation method. Design a mathematical model for solving this problem and use any gradient-based method to find an approximation to the solution of the system (either perform 3 iterations manually or use a Matlab program). (Sample codes implementing Newton’s, steepest descent and conjugate gradient methods can be found at my 3OR website.) Hint: minimise the distance between the right- and left-hand sides.

3. Use the method developed in Q.2 to solve the system

$$(x_1 - 2)^4 + (x_1 - 2x_2)^2 = 5, \quad x_1^2 - x_2 = 0.$$ 

4. Perform 2 iterations of the linear conjugate gradient method (Fletcher-Reeves method) for each of the following problems:

(a) min $f(x) = (x_1 - 2)^4 + (x_1 - 2x_2)^2$ starting from the initial guess $x^0 = (0, 3)^T$.

(b) * min $f(x) = 4x_1^2 + 4x_2^2 - 4x_1x_2 - 12x_2$ starting from $x^0 = (-0.5, 1)^T$.

5. Use Lagrange Multipliers method to solve min $f(x) = x_1 x_2$ s.t. $x_1^2 + x_2^2 = 1$.

6. Solve the following problems using Kuhn-Tucker conditions:

(a) max $f(x) = (x - 4)^2$ subject to $1 \leq x \leq 6$.

(b) *

$$
\begin{align*}
\text{minimize} & \quad -(3x_1 + x_2) \\
\text{subject to} & \quad x_1^2 + x_2^2 \leq 5, \\
& \quad x_1 - x_2 \leq 1.
\end{align*}
$$

(c) max $f(x) = \ln(x_1 + x_2)$ s.t. $x_1 + 2x_2 \leq 5$ and $x_1, x_2 \geq 0$.

(d) min $f(x) = (x_1 - 3)^2 + (x_2 - 3)^2$ s.t. $4 - x_1 - x_2 \geq 0$ and $x_1, x_2 \geq 0$. 