University of Western Australia  
School of Mathematics and Statistics  
MATH1050 Calculus C  
Semester 1, 2010  

Test 3 — Mid-Semester Test  

Student Number: 

Family Name: SOLUTIONS AND MARKING SCHEME  
Use Block Letters  

Given Name: 
Use Block Letters  

Tutorial Day and Time: tick the appropriate box.  

☐ Monday 2pm  
☐ Tuesday 2pm  
☐ Friday 9am  
☐ Friday 11am  

This test counts for 15% of your final mark for MATH1050 Calculus C.  

There are THREE questions in this test, worth a total of 50 marks.  

Attempt all questions.  

Marks are given for clarity and correctness of method, not just for correct answers.  

Appropriate working with reasons should be given.  

Write your answers in the blank space following the question.  

The last page has been left blank for rough working.  

Time Limit: 1 hour
1. Linear Equations (12 marks)

(a) Solve the following equation for \( x \).

\[
\frac{x + 1}{4} + 2 = \frac{6x + 2}{8}
\]

\[
8 \left[ \frac{x + 1}{4} + 2 \right] = 8 \cdot \frac{6x + 2}{8}
\]

\[
\Rightarrow 2x + 2 + 16 = 6x + 2
\]

\[
\Rightarrow 16 = 4x
\]

\[
\therefore x = 4
\]
(b) How many litres of a 70% hydrochloric acid solution should be added to a 20% hydrochloric acid solution to make 4 litres of a 50% acid solution? (4 marks)

<table>
<thead>
<tr>
<th>Vol of Soln</th>
<th>70%</th>
<th>20%</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
<td>4-x</td>
<td></td>
</tr>
<tr>
<td>0.7x</td>
<td>0.2(4-x)</td>
<td>0.5(4)</td>
<td></td>
</tr>
</tbody>
</table>

\[
0.7x + 0.2(4-x) = 0.5(4)
\]

\[
0.7x + 0.8 - 0.2x = 2
\]

\[
0.5x = 1.2
\]

\[
x = \frac{1.2}{0.5} = 2.4
\]

2.4 l of 70% soln
(1.6 l of 20% soln)
(c) An geologist want to determine the weight composition of two laterite nickel ores, A which is limonitic and B which is nontronitic. An assay of 100 kg of type A and 200 kg of Type B yielded 5.5 kg of nickel, while another assay of 200 kg of Type A and 300 kg of Type B yielded 9 kg of nickel. What is the percent composition by weight of each ore type?

Let $x = \% \text{ of Ni in type A}$

$y = \% \text{ of Ni in type B}$

Then

\[
\begin{align*}
100 \cdot \frac{x}{100} + 200 \cdot \frac{y}{100} &= 5.5 \\
200 \cdot \frac{x}{100} + 300 \cdot \frac{y}{100} &= 9
\end{align*}
\]

\[
\Rightarrow \begin{align*}
2x + 4y &= 5.5 \\
2x + 3y &= 9
\end{align*}
\]

\[
R_1 - R_2 : \quad y = 2
\]

\[
\text{From the first equation}, \quad x + 2(2) = 5.5
\]

\[
\Rightarrow x = 1.5
\]

Type A : 1.5 \%

Type B : 2 \%

(5 marks)
2. Factorisations and Quadratics (20 marks)

(a) Simplify

\[
\frac{8x^2 - 10x - 3}{16x^2 - 1} \div \frac{2x^2 - 15x + 18}{4x^2 - x} \times \frac{x^3 - 36x}{3x^2 + 18x}
\]

(5 marks)

\[
= \frac{(4x+1)(2x-3)}{(4x+1)(x-1)} \times \frac{x(x-1)}{(2x-3)(x-6)} \times \frac{x(x^2-36)}{3x(x+6)}
\]

\[
= \frac{x(x+6)(x-6)}{3(x-6)(x+6)}
\]

\[
= \frac{x}{3}
\]
(b) How many solutions does the equation $2x^2 - 3x - 5 = 0$ have? 

\[ \Delta = b^2 - 4ac \]
\[ = (-3)^2 - 4(2)(-5) \]
\[ = 9 + 40 \]
\[ = 49 > 0 \]

so the equation has two solutions.

(c) An explosion is space sends two asteroids travelling in space so that the distance $d(t)$ (in thousands of km) between them at time $t$ (in years) is given by

\[ d(t) = 16t - 3t^2, \quad t \geq 0. \]

How long does it take for the asteroids to first be 16,000 km apart? 

We need to solve

\[ d(t) = 16 \]
\[ 16t - 3t^2 = 16 \]
\[ 3t^2 - 16t + 16 = 0 \]
\[ (3t - 4)(t - 4) = 0 \]
\[ t = \frac{4}{3}, 4 \]

The two asteroids are 16,000 km apart the first time at $\frac{4}{3}$ years.
(d) Let \( y = x^2 - 6x + 8 \). Sketch the graph of \( y \), showing all intercepts and the co-ordinates of the vertex. (4 marks)

\[
y - \text{int: } x = 0 \Rightarrow y = 8
\]

\[
x - \text{int: } y = 0 \Rightarrow x^2 - 6x + 8 = 0
\]
\[
\Rightarrow (x - 4)(x - 2) = 0
\]
\[
\Rightarrow x = 2, 4
\]

**Vertex**: \( x = \frac{2 + 4}{2} = 3 \)

\[
y = 3^2 - 6(3) + 8 = -1
\]

[IF GRAPH CORRECTLY SHOWS IMPORTANT POINTS THEN GIVE CORRESPONDING MARKS FOR WORKING]
(e) Let $y = x^2 - 2x + 3$.

i. Show that $y = (x - 1)^2 + 2$ (1 mark)

\[
y = x^2 - 2x + 3
= x^2 - 2x + 1 + 2
= (x - 1)^2 + 2
\]

ii. Sketch the graph of $y$, showing any intercepts and the co-ordinates of the vertex. (3 marks)

\[\text{x-intercepts:}
\]

\[\text{No x-intercepts.}
\]

\[\text{Vertex: (1, 2)}
\]
3. Exponents and Logarithms (18 marks)

(a) Simplify the following expressing with positive indices:

\[
\left( \frac{8a^6}{b^3} \right)^{\frac{2}{3}} ÷ \sqrt[3]{\frac{64a^6}{b^6}}
\]

(5 marks)

\[
= \frac{2a^2}{b^{-1}} \times \frac{b^{-2}}{4-3}
\]

\[
= \frac{1}{2ab}
\]

(b) Let \( x = \log_4 3 \) and \( y = \log_4 5 \). Express the following in terms of \( x \) and \( y \).

i. \( \log_4 15 \)

(2 marks)

\[
= \log_4 (3 \cdot 5)
\]

\[
= \log_4 3 + \log_4 5
\]

\[
= x + y
\]
ii. \( \log_4 45 \)

\[
= \log_4 (9 \cdot 5) \\
= \log_4 9 + \log_4 5 \\
= \log_4 3^2 + \log_4 5 \\
= 2 \log_4 3 + y \\
= 2x + y
\]  

(3 marks)

iii. \( \log_4 1.25 \)

\[
= \log_4 \left( \frac{5}{4} \right) \\
= \log_4 5 - \log_4 4 \\
= y - 1
\]  

(4 marks)
(c) Solve the following equation for $x$:

$$2^{x+1} = 7$$

\[
\log_{10} 2^{x+1} = \log_{10} 7
\]

\[
\therefore (x+1) \log_{10} 2 = \log_{10} 7
\]

\[
\Rightarrow x + 1 = \frac{\log_{10} 7}{\log_{10} 2}
\]

\[
\therefore x = \frac{\log_{10} 7}{\log_{10} 2} - 1
\]
ROUGH WORKING