Summary of Transformations

We have seen six kinds of Transformations:

1. Vertical Translation (in the $y$–direction)
   $$y = f(x) \pm d.$$  

2. Horizontal Translation (in the $x$–direction)
   $$y = f(x \pm c).$$  

3. Dilation on the $y$–axis
   $$y = af(x), \ SF a.$$  

4. Reflexion on the $x$–axis
   $$y = -f(x).$$  

5. Reflexion on the $y$–axis
   $$y = f(-x).$$  

6. Dilation on the $x$–axis
   $$y = f(bx), \ SF \frac{1}{b}.$$
Summary of Transformations

We have seen six kinds of Transformations:

1. Vertical Translation (in the $y$–direction) $y = f(x) \pm d$.
Summary of Transformations

We have seen six kinds of Transformations:

1. Vertical Translation (in the $y$–direction) $y = f(x) \pm d$.
2. Horizontal Translation (in the $x$–direction) $y = f(x \pm c)$.
We have seen six kinds of Transformations:

1. Vertical Translation (in the $y$–direction) $y = f(x) \pm d$.
2. Horizontal Translation (in the $x$–direction) $y = f(x \pm c)$.
3. Dilation on the $y$–axis $y = af(x)$, SF $a$. 
Summary of Transformations

We have seen six kinds of Transformations:

1. Vertical Translation (in the y–direction) \( y = f(x) \pm d \).
2. Horizontal Translation (in the x–direction) \( y = f(x \pm c) \).
3. Dilation on the y–axis \( y = af(x) \), SF \( a \).
4. Reflexion on the x–axis \( y = f(-x) \).
Summary of Transformations

We have seen six kinds of Transformations:

1. Vertical Translation (in the $y$–direction) $y = f(x) \pm d$.
2. Horizontal Translation (in the $x$–direction) $y = f(x \pm c)$.
3. Dilation on the $y$–axis $y = af(x)$, SF $a$.
4. Reflexion on the $x$–axis $y = -f(x)$.
5. Reflexion on the $y$–axis $y = f(-x)$.
We have seen six kinds of Transformations:

1. Vertical Translation (in the $y$–direction) $y = f(x) \pm d$.
2. Horizontal Translation (in the $x$–direction) $y = f(x \pm c)$.
3. Dilation on the $y$–axis $y = af(x)$, SF $a$.
4. Reflexion on the $x$–axis $y = -f(x)$.
5. Reflexion on the $y$–axis $y = f(-x)$.
6. Dilation on the $x$–axis $y = f(bx)$, SF $1/b$. 
We have seen six kinds of Transformations:

1. Vertical Translation (in the $y$–direction) $y = f(x) \pm d$.
2. Horizontal Translation (in the $x$–direction) $y = f(x \pm c)$.
3. Dilation on the $y$–axis $y = af(x)$, SF $a$.
4. Reflexion on the $x$–axis $y = -f(x)$.
5. Reflexion on the $y$–axis $y = f(-x)$.
6. Dilation on the $x$–axis $y = f(bx)$, SF $1/b$.

$$y = -af(-bx \pm c) \pm d$$
Basic Examples for Transformations

Note the changes in domain and range!

\[ y = x^2 \quad \rightarrow \quad y = x^2 \pm 2 \]
Basic Examples for Transformations

Note the changes in domain and range!

\[ y = x^2 \quad \rightarrow \quad y = x^2 \pm 2 \]

\[ y = \sqrt{x} \quad \rightarrow \quad y = \sqrt{x \pm 2} \]
Basic Examples for Transformations

Note the changes in domain and range!

\[ y = x^2 \quad \rightarrow \quad y = x^2 \pm 2 \]

\[ y = \sqrt{x} \quad \rightarrow \quad y = \sqrt{x \pm 2} \]

\[ y = \sqrt{x} \quad \rightarrow \quad y = 0.5 \sqrt{x} \]
Basic Examples for Transformations

Note the changes in domain and range!

\[ y = x^2 \quad \rightarrow \quad y = x^2 \pm 2 \]

\[ y = \sqrt{x} \quad \rightarrow \quad y = \sqrt{(x \pm 2)} \]

\[ y = \sqrt{x} \quad \rightarrow \quad y = 0.5\sqrt{x} \]

\[ y = x^3 \quad \rightarrow \quad y = -x^3 \]
Note the changes in domain and range!

\[ y = x^2 \quad \rightarrow \quad y = x^2 \pm 2 \]

\[ y = \sqrt{x} \quad \rightarrow \quad y = \sqrt{(x \pm 2)} \]

\[ y = \sqrt{x} \quad \rightarrow \quad y = 0.5\sqrt{x} \]

\[ y = x^3 \quad \rightarrow \quad y = -x^3 \]

\[ y = \sqrt{x} \quad \rightarrow \quad y = \sqrt{-x} \]
Basic Examples for Transformations

Note the changes in domain and range!

\[ y = x^2 \quad \rightarrow \quad y = x^2 \pm 2 \]

\[ y = \sqrt{x} \quad \rightarrow \quad y = \sqrt{(x \pm 2)} \]

\[ y = \sqrt{x} \quad \rightarrow \quad y = 0.5\sqrt{x} \]

\[ y = x^3 \quad \rightarrow \quad y = -x^3 \]

\[ y = \sqrt{x} \quad \rightarrow \quad y = \sqrt{-x} \]

\[ y = \sin(x) \quad \rightarrow \quad y = \sin(2x) \]
Transformations are the first kind of function compositions we learned. Consider:

\[ f(s) = s^2, \]
Transformations are the first kind of function compositions we learned. Consider:

\[ f(s) = s^2, \]
where \( s = g(x) = x + 1 \)
3.4 Function Composition

Transformations are the first kind of function compositions we learned.
Consider:

\[ f(s) = s^2, \]

where \( s = g(x) = x + 1 \)

\[ \iff f(g(x)) = f(x + 1) = (x + 1)^2 \]
Transformations are the first kind of function compositions we learned.
Consider:

\[ f(s) = s^2, \]
where \( s = g(x) = x + 1 \)

\[ \iff f(g(x)) = f(x + 1) = (x + 1)^2 \]

Notation:

\[ f \circ g(x) = f(g(x)). \]
Domain and Range of composed functions

- **Input for Domain of**
  - Domain of $g(x)$

- **Output of Range of**
  - Range of $f(s)$
Domain and Range of composed functions

\[ g(x) \]

- Input for Domain of \( g(x) \)
- Output of Range of \( g(x) \)
- Input for Domain of \( f(s) \)
Domain and Range of composed functions

- **Input for Domain of** $g(x)$
- **Output of Range of** $g(x)$
- **Input for Domain of** $f(s)$
- **Output of Range of** $f(s)$
Examples

Draw the following functions and determine their domain and range:

\[ f(s) = s + 1, \quad \text{where } s = g(x) = x^2 \]  \hspace{1cm} (1)

\[ f(s) = \sqrt{s}, \quad \text{where } s = g(x) = -x \]  \hspace{1cm} (2)

\[ f(s) = \frac{1}{s}, \quad \text{where } s = g(x) = \sqrt{x} \]  \hspace{1cm} (3)

\[ f(s) = \sqrt{s}, \quad \text{where } s = g(x) = x^2 \]  \hspace{1cm} (4)