4. Indices and Logarithm
4.2 Exponential Functions

Consider

\[ f(x) = a \times b^x. \]

Such functions are called exponential functions, where

- \( a \) is a coefficient.
- \( b \) is the base.
- and the variable \( x \) is an exponent.
Some characteristics of exponential functions $f(x) = b^x$:

- $b$ has to be $> 0$.
- $f(x)$ will intersect the $y$–axis at 1 for $x = 0$.
- for $b > 1$: $f(x)$ increases for $x$ increasing.
- for $b < 1$: $f(x)$ decreases for $x$ increasing.
- Domain $D_f = (-\infty, \infty)$.
- Range $R_f = (0, \infty)$.
- Indices laws can be used for calculations involving several exponential functions.
Examples:

\[ f(x) = 2^x \]  \hspace{1cm} (1)

\[ f(x) = e^x \]  \hspace{1cm} (2)

\[ f(x) = 10^x \]  \hspace{1cm} (3)

\[ f(x) = (e^x)^2 \]  \hspace{1cm} (4)

\[ f(x) = e^x \times e^{-x} \]  \hspace{1cm} (5)

\[ f(x) = e^x \times e^{3x} \]  \hspace{1cm} (6)

\[ f(x) = e^x \times e^{3x} \]  \hspace{1cm} (7)
4.2 Logarithmic function

Consider the inverse function of an exponential function:

\[ f(x) \]

\[ f^{-1}(x) \]
The inverse function of an exponential function is called:

THE LOGARITHMIC FUNCTION.
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Consider $f(x) = b^x$. 
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Consider \( f(x) = b^x \).
\( f^{-1}(x) = \log_b(x) \).
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Consider \( f(x) = b^x \).

\[ f^{-1}(x) = \log_b(x). \]

We say: The logarithm of \( x \) to base \( b \).
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Consider \( f(x) = b^x \).
\[
f^{-1}(x) = \log_b(x).
\]
We say: The logarithm of \( x \) to base \( b \).

It follows that

\[
f^{-1} \circ f(x) = f^{-1}(b^x) = \log_b(b^x) = x
\]