Attempt questions 2 (a), (h), Q3 (a), (i), (n) and (o) before your tutorial class in week 4 starting 15 March.

A. Revision of Mixture Models

1. A laboratory stores three concentrations of HCl—10%, 50% and 90%. For a particular experiment, the Professor orders the following:
   (a) 1 L of 30% solution,
   (b) 2.5 L of 60% solution, and
   (c) 3 L of 80% solution.

   Write a prescription informing the laboratory assistant how to prepare these solutions.

B. Simultaneous Equations

2. Solve the following set of simultaneous equations.
   
   (a) \(2x + y = 8\)
   \(3x - y = 7\)
   
   (b) \(2x + 3y = 19\)
   \(4x - y = 3\)
   
   (c) \(2x + 3y = 8\)
   \(3x - 2y = -7\)
   
   (d) \(x + 2y = 10\)
   \(x - y = 3\)
   
   (e) \(4x - y = 0\)
   \(2x - y = -2\)
   
   (f) \(2x + y = 4\)
   \(5x + 2y = 9\)
   
   (g) \(x + 3y = 3\)
   \(-2x + y = 8\)
   
   (h) \(2x - 3y = 5\)
   \(5x + 2y = -16\)
   
   (i) \(x + y = 5\)
   \(2x + y = 7\)
   
   (j) \(4x + y = 10\)
   \(2x + 3y = 10\)

C. Modelling using Simultaneous Equations

3. In the following problems, first formulate an appropriate set of simultaneous equations and then solve them.
   
   (a) How many litres of a 10% solution of phosphoric acid should be added to 20 litres of a 60% solution to obtain a 50% solution? All percentages are by volume.
   
   (b) Two unblended manganese ores contain 40% and 25% by weight respectively of manganese. How many tonnes of each must be mixed to give 100 tonnes of blended ore containing 35% manganese?
   
   (c) One thousand litres of milk testing 4% butterfat are to be reduced to 3%. How many litres of cream testing 23% butterfat must be separated from the milk to produce the required result? All percentages are by volume.
   
   (d) Tank A contains a mixture of 10 litres of water and 5 litres of frother. Tank B contains 12 litres of water and 3 litres of frother. Both tanks have been thoroughly stirred. How many litres should be taken from each tank and combined to obtain an 8 litre solution containing 25% frother by volume? (Frothers are used in mining to float minerals.)
(e) A laboratory stores sulphuric acid in concentrations of 10% and 30%. A particular experiment
requires 10 litres of 15% acid. How much of the two available solutions should be mixed to obtain
the required solution?

(f) Tickets for an ice-skating display are sold at $5 for adults and $2 for children. If 101 tickets were
sold altogether for a take of $394, find the number of adults and children who attended.

(g) The sum of two numbers is 28 and their difference is 12. Find the numbers.

(h) The ancient Roman emperor Augustus was fond of gold and silver sovereigns. By royal decree
every gold sovereign was to weigh 50 g and every silver one to weigh 40 g. This year a new
jeweller was hired to prepare the sovereigns. Augustus suspects that the jeweller is cheating him
and delivering sub-weight sovereigns. The punishment for cheating the Emperor is death. The
first consignment contained thirty gold and twenty silver sovereigns and together weighed 2,250
g, and the second consignment of fifteen gold and 25 silver sovereigns weighed a total of 1,550 g.
Will the jeweller live?

(i) The law connecting friction $F$ and load $L$ for an experiment is of the form $F = aL + b$ where $a$
and $b$ are constants. When $F = 5.6$, $L = 8.0$ and when $F = 4.4$, $L = 2.0$.
   i. Find the values of $a$ and $b$.
   ii. Find the force when the load is 6.5.

(j) An ostrich lays either brown or speckled eggs. Five brown eggs and two speckled ones weigh 8
kg, and four brown eggs and five speckled ones weigh 11.5 kg.
   i. Find the weight of each type of egg.
   ii. An ostrich nest contains six brown and two speckled eggs. What is the total weight of the
   eggs?

(k) An island contains foxes and rabbits. An ecologist keeps counts of both species to study their
interaction and how their populations change over time. Last year she found that the total number
of foxes and rabbits was 7,290 and the foxes were only 1 eighth of the rabbit population. How
many of each species was present?

(l) Mei Li has $400 to buy some pastries for a party. If she buys 10 apple crumbles and 50 muffins
she will underspend by $100. If she buys 15 apple crumbles and 70 muffins she will overspend by
$40.
   i. What is the cost of each of the items?
   ii. If she want to buy one muffin for each of her 80 guest, how many apple crumbles can she
   buy?

(m) A school takes three classes for ice-skating. The first class of twenty students and three teachers
costs $375 and the second class of fifteen students and four teachers pays $325. What is the cost
for the third class consisting of twelve students and two teachers?

(n) Two types of flooring tiles are used by a builder for a house. The smaller of the tiles have area
25cm$^2$ each and the larger ones have area 40cm$^2$ each. Further, the smaller tiles cost $1 each and
the larger ones cost $1.50 each. In a particular house a total of 400 tiles are used to cover a floor
area of 13,750cm$^2$. What is the total cost of the tiles?

(o) A plane flies from Perth to Sydney in three and a half hours and from Sydney to Perth in five and a
half hours. The wind direction in always from Perth to Sydney (the Fremantle Doctor). The air
distance between Perth and Sydney is 3,300 km. Assuming that the aircraft has the same true
airspeed, what is the wind speed of the Fremantle Doctor?

(p) It is supposed that the systolic blood pressure (SBP, measured in mm Hg) in children is related
to height ($H$, in cm). A linear regression model is fitted to the data, so that $SBP = a + bH$
where the constants $a$ and $b$ are computed using data. Measurements on 100 children gave the
following for $a$ and $b$:

\[
\begin{align*}
    a + 120b &= 110, \\
    12a + 1400b &= 1300,
\end{align*}
\]
   i. Solve the equations to find the value of the co-efficients $a$ and $b$.
   ii. Using these values write down the equation relating the systolic blood pressure to height.
   iii. What is the difference between the systolic blood pressures of two children who are 1.2 m
and 1.3 m tall respectively?