3A7 Dynamical Systems and Chaos

(MATH3327)

1. Introduction
3A7 Dynamical Systems and Chaos

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What is a dynamical system?

A complex system of interacting things whose configuration, or relationship to each other, changes with time.

Dynamical System n.
complex whole
connected things, or parts
organised body or material or immaterial things
active; potent; energetic
of motive force (opp. static)
of force in actual operation (opp. potential)
What is a dynamical system?

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System n.
- complex whole
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- organised body or material or immaterial things

A complex system of interacting things whose configuration, or relationship to each other, changes with time.
What is a dynamical system?
What is a dynamical system?

YOU!
Reality is everything that is

**Reality** is everything that is
Reality and Models

Reality is everything that is.
All that we know of Reality is from Observations.
Reality and Models

- *Reality* is everything that is
- All that we know of *Reality* is from *Observations*
- We construct *Models of Reality* using *Observations*
Examples of Observations
Examples of Observations
Examples of Observations

- Disease dynamics.
- Heart and breath rhythm.
- Population dynamics.
- Electronics.

...
A warning!

Normal Mathematics:

Axiom \implies \text{Theorem} \implies \text{Proof} \implies \text{Example}

This unit is different!
Focus on generic concepts / ideas
A warning!

Normal Mathematics:
Axiom
A warning!

Normal Mathematics:
Axiom $\implies$ Theorem
A warning!

Normal Mathematics:
Axiom ⇒ Theorem ⇒ Proof
A warning!

Normal Mathematics:
Axiom ⇒ Theorem ⇒ Proof ⇒ Example
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Normal Mathematics:
Axiom ⇒ Theorem ⇒ Proof ⇒ Example

This unit is different!
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Normal Mathematics:
Axiom ⇒ Theorem ⇒ Proof ⇒ Example

This unit is different!

Focus on generic concepts / ideas
What is Chaos?

The behavior of chaotic systems appears to be random. This happens even though these systems are deterministic, meaning that their future dynamics are fully defined by their initial conditions with no random elements involved. This behavior is known as deterministic chaos, or simply chaos.
Essence of Chaos

- **State and dynamics:**
  - **State:** one number
  - **Dynamical rule:** Square the number, then subtract from number you first thought of
  - **Mathematically:**
    \[ x_{t+1} = a - x_t^2 \]
Essense of Chaos

- **State and dynamics:**
  - **State:** one number
  - **Dynamical rule:** Square the number, then subtract from number you first thought of
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    \[ x_{t+1} = a - x_t^2 \]

- Results look like this, when \( a = 1.99 \)
Essence of Chaos

- **State and dynamics:**
  - **State:** one number
  - **Dynamical rule:** Square the number, then subtract from number you first thought of
  - **Mathematically:**
    \[ x_{t+1} = a - x_t^2 \]
  - Results look like this, when \( a = 1.99 \)

- Really there are two numbers here:
  - A variable or **state** \( x_t \)
  - A fixed **parameter** \( a \)
Dynamics for different $a$

$a = 0.5$

Graph showing dynamics for $a = 0.5$, indicated by a steady state pattern.
Dynamics for different $a$

$a = 0.5$

$a = 1$
Dynamics for different $a$

$a = 0.5$

$a = 1$

$a = 1.5$
Dynamics for different $a$

$a = 0.5$
steady state

$a = 1$

$a = 1.5$
Dynamics for different $a$

$a = 0.5$
steady state

$a = 1$
periodic

$a = 1.5$
Dynamics for different $a$

$a = 0.5$
- Steady state

$a = 1$
- Periodic

$a = 1.5$
- Chaotic
Attractor versus Parameter $a$
Attractor versus Parameter $a$
Crisis and Hysteria
Crisis and Hysteria

Beware, Bifurcations!
Crisis and Hysteria

Beware, Bifurcations!
Crisis and Hysteria

Beware, Bifurcations!
Crisis and Hysteria
Crisis and Hysteresis

Beware, Bifurcations!
Information about 3A7

Web pages

See this page for credit, timetable, content, assessment, prerequisites.

See this page for announcements, lecture notes, question sheets, assignments, and solutions question sheets.
Assessment

Assessment will comprise an exam worth 70% and continuing assessment of 30% made up of assignments and possibly one or two short tests.
You should also be aware of the University policies on plagiarism.
Calculators will be permitted in exam and tests, but are probably of little help.
There is not a text book for the unit, however, useful references are:


These books may have more recent editions. The library has many shelves of books on dynamical systems and chaos. Many of these will also provide suitable treatments of material covered in this unit.

Outline

- Introduction:
  - What is a dynamical system?
  - Historical development
  - Discovery of Chaos
  - Importance of nonlinearity
  - Stability and instability
  - Modelling, uncertainty and prediction
  - Applications
Outline

- Differential equations:
  - Pictures of flows.
  - Fundamental Theorem of Flows.
  - Gronwall inequality
  - Fixed points and Stability analysis
  - Hartman-Grobman Theorem
  - Saddle-node bifurcation
  - Periodic orbits
  - Hopf bifurcation
  - Poincaré-Bendixson Theorem
  - Lorenz equations and chaos
Outline

Maps
- Poincaré maps
- Baker’s map
- Symbolic dynamics
- 1D maps
- Period doubling bifurcation
- Lyapunov exponents
- 2D maps (Horseshoe maps)
Climate Change and Chaos
What is Climate Change?

- Weather is not random
- Complex ever-evolving patterns
- Dynamic patterns described by mathematics
- Weather forecasting is Science’s most ambitious game
- Climate change, however, is a whole new ball game
Mathematics of Change

- A little history of the mathematics of change
- A little history of weather forecasting
- Chaos and limits to forecasting
- Chaos and climate change
Change & Chaos
In the beginning ...

circa 3000BC
Forecasting Seasons
Forecasting Seasons
Forecasting Seasons

Winter

Summer

E

N

S

W
Forecasting Seasons
Forecasting Seasons

Differential Heating
Nicholas Copernicus
(1473–1543)

- Heliocentric model
  - Earth is not fixed centre
  - Rotates on axis
  - Revolves around sun
Nicholas Copernicus  
(1473–1543)

- Heliocentric model
  - Earth is not fixed centre
  - Rotates on axis
  - Revolves around sun

Earth moves in 2 ways
The motion of a planet is an ellipse with the Sun at one focus. The chord between the Sun and the planet sweeps out equal area in equal times. As good as it gets without calculus.
The motion of a planet is an ellipse with the Sun at one focus.

The chord between the Sun and the planet sweeps out equal area in equal times.

As good as it gets without calculus.
The motion of a planet is an ellipse with the Sun at one focus.

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The motion of a planet is an ellipse with the Sun at one focus.

The *chord* between the Sun and the planet sweeps out equal area in equal times.

As good as it gets without calculus.
Johannus Kepler
(1571–1630)

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Elliptical orbits
Isaac Newton
(1643–1727)

- Three Laws of Motion
- Law of Gravity
- Calculus
Isaac Newton
(1643–1727)

- Three Laws of Motion
- Law of Gravity
- Calculus
- Notion of state
  - Quantities or variables
  - Rules or dynamics
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- Three Laws of Motion
- Law of Gravity
- Calculus
- Notion of state
  - Quantities or variables
  - Rules or dynamics

π
State & Dynamics
Pierre-Simon Laplace
(1749–1827)

- Theory of Probability
- Celestial Mechanics
Pierre-Simon Laplace
(1749–1827)

- Theory of Probability
- Celestial Mechanics
- Determinism
  *(Clockwork Universe)*
Laplace’s Daemon

A vast intellect who at a certain moment

Knows the position and motion of all particles,

(Dynamics)

And possesses the capacity to calculate them

Can compute all past and future

This is the basis of Science and Forecasting
Laplace’s Daemon

A vast intellect who at a certain moment

- Knows the position and motion of all particles, \((\text{State})\)
Laplace’s Daemon

A vast intellect who at a certain moment

- Knows the position and motion of all particles, \textit{(State)}
- Knows the laws that govern their interaction, \textit{(Dynamics)}
Laplace’s Daemon

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Can compute all past and future
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- And possesses the capacity to calculate them

Can compute all past and future

This is basis of Science and Forecasting
There is problem ...
James Clerk Maxwell
(1813–1879)

- There is problem ...
- Sensitivity to initial conditions
There is a problem ...

Sensitivity to initial conditions

Butterfly effect
Is the solar system stable?

Almost discovers chaos
Jules Henri Poincaré (1854–1912)

• Is the solar system stable?
• *Almost* discovers chaos
Orbital variation

Eccentricity

Obliquity

Earth moves again!
Orbital variation

Eccentricity

Obliquity

Eccentricity (≈ 100ka)

Obliquity (≈ 41ka)
Orbital variation

Eccentricity (≈ 100 ka)

Obliquity (≈ 41 ka)

Earth moves again!
Weather & Chaos

Short Version
Francis Galton
(1822–1911)

- Explorer
- Scientist
- Statistician
Francis Galton
(1862)

Statistical approach to forecasting

Find a previous map that looks like this.
Francis Galton

(1862)

Statistical approach to forecasting

Find a previous map that looks like this.
Need to collect maps for $10^4$–$10^{10}$ years.
George Gabriel Stokes
(1819–1903)

- Fluid Mechanics
- Navier-Stokes Equation
Navier-Stokes Equations

\[ \frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{1}{\rho} \nabla P - 2\Omega \times u + \nu \nabla^2 u + f \]

Winds, temperature, humidity, pressure
Sea surface temperature, soil moisture, dust, \( CO_2 \)
Lewis Fry Richardson
(1881–1953)

- Proposed numerical solution of Navier-Stokes equation to forecast weather
- Made the first numerical calculations
Lewis Fry Richardson (1911)

64,000 people needed to compute a weather forecast.
Lewis Fry Richardson
(1911)

64,000 people needed to compute a weather forecast.
More like 2.5 billion people.
... and now

Large expensive standing stones
Edward Lorenz
(1917–2008)

- Maxwell’s idea and Poincaré’s work not fully appreciated
- Convection of fluids
- Deterministic non-periodic flows
The Lava Lamp

- Gentle heating
- Stronger heating
Chaos

- Complex behaviour from simple dynamical rules
- Small initial error grows exponentially until it is large
- Try this at home: $x^2, \ln, x^2, \ln, x^2, \ln, ...$
Sensitivity of nonlinear systems

Air moving across the ocean meets an island, what will it do?

Go over it? Go round it?

Detailed weather forecasts beyond two weeks are impossible
Limited observations

Every 6, 12 or 24 hours release a balloon to obtain a profile. Satellites provide lots of data, but little is of immediate use.
Ensemble forecasting

December 25, 1999 storm over Europe
What is Truth?

*Reality* is everything that is. All that we know of *Reality* is from *Observations*. We construct *Models* of *Reality* using *Observations* and they are not the same as *Reality*. There is *no Truth*!
Crisis and Hysteresis

Beware, Bifurcations!
Climate & Chaos
Energy Budget
Energy Budget

Insolation

Short wavelength

Albedo

Clouds
Energy Budget

Insolation

Short wavelength

Albedo

Clouds

Long wavelength
Insolation

Energy Budget

Greenhouse gases:
- H₂O
- CO₂
- NH₃
- NO₃

Short wavelength:

Albedo

Long wavelength:

Clouds
Four Causes of Climate Change

- **Insolation**
- **Albedo**
- **Greenhouse Gases**
- **Dynamics**
Four Causes of Climate Change

- **Insolation**
  - Energy coming in

- **Albedo**
  - Energy reflected back

- **Greenhouse Gases**
  - Blanket that traps heat

- **Dynamics**
  - Redistribution of energy
Four Causes of Climate Change

- **Insolation**
  - Energy coming in
    - Solar activity
    - Orbital variation

- **Albedo**
  - Energy reflected back
    - Snow and Ice sheets
    - Clouds

- **Greenhouse Gases**
  - Blanket that traps heat
    - Natural sources (CO$_2$: Atmos. 597Gt, Oceans 37100Gt)
    - Human contribution (CO$_2$: Atmos. 27.6%, Oceans 0.27%)

- **Dynamics**
  - Redistribution of energy
    - Chaos
    - Bifurcations
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- **Dynamics**
  - Redistribution of energy
    - Chaos
    - Bifurcations
Sediment and Ice Cores
Forminifera, bubbles, and isotopes
Paleoclimate from 500Ma

500 million years of climate change (sediment cores)
Paleoclimate from 500Ma

500 million years of climate change (sediment cores)
Paleoclimate from 500Ma

500 million years of climate change (sediment cores)
Paleoclimate from 65Ma

65 million years of climate change (sea bed sediments)
Generally decreasing $CO_2$ from 50Ma
Paleoclimate from 65 Ma

65 million years of climate change (sea bed sediments)
Alligators and Swamps near North Pole
Paleoclimate from 65Ma

65 million years of climate change (sea bed sediments)
Run-away cooling from increased ice and snow albedo
Paleoclimate from 65Ma

65 million years of climate change (sea bed sediments)
Stable for 10Ma, then mysterious thawing
Paleoclimate from 65Ma

65 million years of climate change (sea bed sediments)
Stabilisation at previous level
Paleoclimate from 65Ma

65 million years of climate change (sea bed sediments)
North pole finally freezes over
Paleoclimate from 65Ma

65 million years of climate change (sea bed sediments)
Focus on last 5Ma
Paleoclimate from 5Ma

5 million years of climate change (sea bed sediments)
Stable climate in 3–5Ma period
Then cooling and oscillations (41ka, 4 degree; 100ka, 8 degree)
Period of cycles changes about 1Ma
Paleoclimate from 5Ma

5 million years of climate change (sea bed sediments)
Stable climate in 3–5Ma period
Then cooling and oscillations (41ka, 4 degree; 100ka, 8 degree)
Period of cycles changes about 1Ma
Paleoclimate from 650ka

650 thousand years of climate change (ice cores)
Large global 100ka cycles of fast warming, slow cooling
Orbital variation

Eccentricity

Obliquity

Eccentricity ($\approx 100$ ka)

Obliquity ($\approx 41$ ka)
Milankovitch Forcing

Milankovitch forcing is seems correlate
But not exactly ...
Relative Heating by Latitude

Maximum and minimum annual heating in last 1Ma

Small variation in heating, largest around 65–80 degrees
Effective Forcing

Heat variation is not enough by itself
Requires dynamical amplification and hysteresis bifurcations
Sensitive Zones?

Forcing most significant between 65–80 degrees
Dynamics with hysteresis
Run-away freezing and thawing.
Interaction of insolation, albedo, GHG, and dynamics.
Paleoclimate from 650ka

650 thousand years of climate change (ice cores)
Large global 100ka cycles of fast warming, slow cooling
Paleoclimate for 30–60ka ago

30–60 thousand years of climate change (ice cores)
Complex chaotic fluctuation in the range 1–5ka
Unlikely to be induced by insolation
Paleoclimate from 650ka

650 thousand years of climate change (ice cores)
Large global 100ka cycles of fast warming, slow cooling
100 thousand years of climate change (ice cores)
Rapid rise to steady state similar to 3–5Ma ago.
Bi-stability: deep glacial chaos and a warm “steady” state
Climate Story So Far ... 

- 5Ma ago, stable temperature about 2 degrees warmer
- 3Ma ago, 41ka, 4 degree, oscillation
- 1Ma ago, 100ka, 8 degree, oscillation
- Correlates with orbital variations, but ...
- Must be amplified by Albedo, GHGs, and chaotic dynamics
- Deep glacial include 2 degree chaotic dynamics
- These oscillations unlikely to be induced by insolation
- Variation of solar constant cannot more than Milankovitch
- Likely bi-stability: Glacial chaos and present day warmth
- These are well understood properties of chaotic dynamics
- But climate mechanisms are not fully understood
Climate Story So Far ... 

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Four Causes of Climate Change

- **Insolation**
  - Energy coming in

- **Albedo**
  - Energy reflected back

- **Greenhouse Gases**
  - Blanket that traps heat

- **Dynamics**
  - Redistribution of energy
Four Causes of Climate Change

- **Insolation**
  - Energy coming in
  - Yes, but only indirectly through dynamical amplification

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    - Run-away cooling and warming

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    - Needed for run-away warming

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- **Dynamics**
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  - Yes.
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- **Dynamics**
  - Redistribution of energy
    - Yes.
    - Amplifies small forcing
Four Causes of Climate Change

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    - Yes
    - Run-away cooling and warming

- **Greenhouse Gases**
  - Blanket that traps heat
    - Yes
    - Needed for run-away warming

- **Dynamics**
  - Redistribution of energy
    - Yes.
    - Amplifies small forcing
    - Bifurcations and hysteresis are important
The last 2000 years
Paleoclimate from 650ka

Likely bi-stability: Glacial chaos and present day warmth
Last 200 years is something very different ...
Paleoclimate from 650ka

Likely bi-stability: Glacial chaos and present day warmth
Last 200 years is something very different ...
Climate of last 2000 years

Northern Hemisphere (historical records, tree rings, etc)
All temperature indicators are now increasing
Is this natural or manmade?
Climate of last 2000 years

Northern Hemisphere (historical records, tree rings, etc)
All temperature indicators are now increasing
Is this natural or manmade?
Sunspots numbers and $^{14}C$ index
Sunspots

Sunspots numbers and $C^{14}$ index
Extended $C^{14}$ index
Sunspots

Overlay $C^{14}$ index on temperature records
Four Causes?

- Insolation
- Albedo
- Greenhouse gases
- Dynamics
Climate Forecasts

SRES Mean Surface Warming Projections

Global surface warming (°C)

-1.0
0.0
1.0
2.0
3.0
4.0

Year
1900 2000 2100 2200 2300

A2
A1B
B1
Constant composition commitment
20th century

Do nothing
Dither and delay
Be reasonable
Stop everything!

Model forecasts
Paleoclimate from 650ka

Likely bi-stability: Glacial chaos and present day warmth
Last 200 years is something very different ...
Paleoclimate from 650ka

Likely bi-stability: Glacial chaos and present day warmth
Last 200 years is something very different...
Paleoclimate from 5Ma

Last time this happened was 3 million years ago
Solar forcing is no longer a factor
Easily 3m sea level rise from Greenland ice sheet melt
Paleoclimate: 65Ma

Could 2–3 degrees of warming thaw Antarctica?
This would raise sea levels 25m
Climate Change Challenge

- A problem science has never faced before
Climate Change Challenge

- A problem science has never faced before
- We cannot make experiments on the climate
Climate Change Challenge

- A problem science has never faced before
- We cannot make experiments on the climate
- We must rely entirely on mathematical models
Risk and Economic Impact?

There are doubts. But sound physical principles. Is dithering and denial worth the risk? Economies remake themselves all the time. No one freezes in the dark...
Risk and Economic Impact?

- There are doubts
- But sound physical principles
- Is dithering and denial worth the risk?
- Economies remake themselves all the time
- No one freezes in the dark ...
Things always change ...
Things always change ...
Risk and Economic Impact?

- There are doubts
- But sound physical principles
- Is worth the risk?
- Economies remake themselves all the time
- No one freezes in the dark ...