1. The quadratic equation $x^2 - 3x - 5 = 0$ has roots $\alpha, \beta$. Determine $\alpha^2 + \beta^2$ and $\alpha^{-2} + \beta^{-2}$.

2. The quadratic polynomial $x^2 + 4x - 1$ has zeros $\alpha, \beta$. Determine $\alpha^3 + \beta^3$ and $\alpha^{-3} + \beta^{-3}$.

\textit{Hint.} $(\alpha + \beta)^3 = \alpha^3 + \beta^3 + 3\alpha^2\beta + 3\alpha\beta^2 = \alpha^3 + \beta^3 + 3\alpha\beta(\alpha + \beta)$.

3. Solve $2 \left( x + \frac{1}{x} \right)^2 - \left( x + \frac{1}{x} \right) = 10$.

4. Use the Remainder and Factor Theorems to factorise
   
   (i) $x^3 - 2x^2 - 5x + 6$ 
   (ii) $x^3 - 5x^2 + 3x + 1$

5. The quadratic polynomial $ax^2 + bx - 4$ leaves remainder 12 on division by $x - 1$ and has $x + 2$ as a factor. Find $a, b$ and the zeros of the polynomial.

6. Find a quadratic equation with roots $2 + \sqrt{3}$ and $2 - \sqrt{3}$.

7. June solved a quadratic equation of the form:

   $$ax^2 + bx + c = 0$$

   and got 2 as a root. Kay switched the $b$ and the $c$ and got 3 as a root. What was June’s equation?

8. The equation $x^2 + ax + (b + 2) = 0$ has real roots. What is the least value that $a^2 + b^2$ could be?

9. If $a, b$ are odd integers, prove that the equation

   $$x^2 + 2ax + 2b = 0$$

   has no rational roots.